# Description of the main research directions investigated by the Institute

The main research directions cover a wide range from mathematics, theoretical computer science, artificial intelligence, statistical data analysis, signal and image processing, and econometrics, to applied areas such as creation of specialised software, as well as hardware design. Many outcomes of our research have found applications in industry and medicine and have led to joint applied projects and commercial solutions.
Some research directions of the Institute have a long tradition, having been investigated for many decades (control theory, pattern recognition, and statistical data analysis), while some ofthe others reflect recent developments and challenges, such as neural networks and deep learning, to name just one example.
We go through the main research directions below. They do not exactly reflect the current organisational structure of the Institute; many of them are investigated jointly by several teams, thus benefiting from their complementary expertise.

## Multidimensional signal processing, analysis and synthesis

This wide research area spans five departments; each of them contributes in specific sub-areas according to its expertise. Our activities in this field range from pure theoretical research to end-user software solutions.

*Image and signal restoration*. The goal is to remove or diminish signal degradation such as blur and noise, or to perform a blind separation of mixed signals. From a mathematical point of view, this is an ill-posed inverse problem that appears in many applications such as microscopy, medicine, surveillance, and biology. We focus on learning-based approaches, variational methods, Bayesian framework and regularised solutions. We have implemented some of the algorithms on powerful hardware platforms such as FPGA or the built-in signal processing unit of a camera.

*Moment theory and invariants*. In image analysis, the design of invariant descriptors is one of the main challenges. We focus on moment invariants of various types (affine invariants, blur and colour invariants, Zernike and Gaussian-Hermite invariants). The major achievement in the area of moment invariants is the theory of projection operators and invariants to image blurring, which can be implemented in various image recognition systems.

*Algorithms for vector and tensor fields.* Vector fields represent a special kind of multidimensional data. In each pixel, the field is assigned to a vector or a matrix which shows the direction and the magnitude of the quantity to be measured. Vector fields appear in numerous scientific and engineering areas. We have proposed two groups of pattern invariants, which can be used for detection of vortices, saddle points and other singularities. Another type of generalisation of vector fields is represented by tensor fields, in which atensor (matrix) is assigned to each voxel.

*Medical imaging.* Many of our theoretical results can be adapted to be immediately used in medical imaging. We have focused on the analysis of videokymographic data (vocal cord investigation), ultrasound images where specific noise-suppression methods are required, and on CT/MRI imaging, where the problem of precise 3D reconstruction arises. The latest

direction concerns diffusion tensor imaging (DTI), where the data is a tensor field showing conductivity of nerve fibres in the human body.
*Image forensics.* In this area**,** we have adapted image restoration methods to an application in criminal evidence detection. Development of methods for forgery identification in video data and approaches for archiving and tracing of possible image/video manipulation represents another key challenge. We have also developed methods that allow us to identify the camera which captured a particular photo or video.

*Visual data modelling*. The emphasis is being put on physically correct visualisation of material appearance, namely in the case of BTF-textured surfaces, which are very difficult to model. The research in this field benefits from a unique piece of hardware equipment – a reflectometer – which allows precise measurements for system training and testing. Visual appearance models of shape and surface of the material are mainly applied in the automobile industry, architecture, and virtual reconstruction of cultural heritage objects.

**Robotics**

*Navigation and visual systems*. This area benefits from the results achieved in image andvideo analysis and recognition. We have developed methods for information fusion andrelated classifiers, object recognition under real-time conditions, and deep-learning based face recognition methods, which can be used in a human-robot interaction.

*Robotic walking.* This research direction has ranged from theoretical development based on classical mechanics system modelling and control up to experimental laboratory research based on building a "walking like platform" consisting of two legs with a torso actuated at ankles and hips. Control design is based on the notion of the so-called virtual holonomic constraints and input-output exact feedback linearisation. The new and more specific types of constraints have been introduced, namely, collocated virtual holonomic constraints. Furthermore, the methods for the estimation and identifications of the underactuated walking robots have been studied. The main focus is on simplified two-dimensional models, addressing only forward dynamics in sagittal plane. Nevertheless, the natural lateral stability for the future possibly three-dimensional walking models has been studied as well.

**Decision making and classification**

*Statistical classification*. In this area, we have achieved new theoretical results in probabilistic neural networks, statistical feature selection, mixture models, and Bayesian classifiers. We have developed recursive Bayesian clustering algorithms and their software implementation in an open source and free environment for engineering computations. The main emphasis hasbeen placed on combinations of various types of the pointer model and mixture componentsto cover as many types of specific data as possible, not limited to the normality assumption. The proposed clustering algorithms have the unified form, allowing us the choice of prior probability density functions and their reproducible statistics updates depending on the component type, which facilitates a further extension of the methodology to additional typesof data.

*Fully probabilistic design of decision strategies,* invented by one of our teams, extends theoryof dynamic, expected-utility-based decision making. It has opened a way to

a quantum-probability version of FPD. This direction is connected with the Decentralised dynamic decision making under uncertainty, where a distributed cooperation of imperfect decision makers having multiple DM goals has been further developed. This has allowed us to achieve an automatic harmonisation of preferences within a fully scalable flat cooperation scheme.

*Decision making in economy*. In this quickly developing area, we have focused on new models for the dynamic risk measurement, optimal decision making and advanced assetpricing techniques, measurement of dynamic networks and their connectedness, as well as interactions between the financial sector and the real economy. These models help us better understand and explain the complex changes in behaviour and decision making of economic agents. The research has direct implications on policy makers, as well as practitioners. We have developed mathematical models as well as statistical methodologies for understanding economic and financial problems, with the main focus on estimation in real-world data. Special attention has been devoted to analysing high-dimensional data sets (big data) and the usage of machine learning methods in economics and finance. The main emphasis of the workin the fields of financial econometrics and energy economics is hence placed on the development of new econometric techniques for modelling data from financial and energy markets, and testing beliefs about price formation in these markets.

*Logic.* We concentrate on abstract algebraic logic, the subfield of mathematical logic that studies logical systems by considering the ways in which they may be associated to their algebraic counterparts. We have considered a very general family of propositional logics with a reasonable notion of implication and their interplay with a generalised disjunction satisfyingthe proof-by-cases property and we have obtained several useful characterisations of semilinearity (the essential property of fuzzy logics) in terms of disjunctions. We have also characterised completeness properties of logical systems with respect to general algebraic semantics. We have proposed a novel extension of abstract algebraic logic to infinitary logics,i.e., systems with infinitary rules. Furthermore, we have been involved in the non-classical predicate logics *and their model theory* (model theory of non-classical predicate logics, homomorphisms, generalisation of Löwenheim-Skolem theorems). In fuzzy logic, we have focused on systems with higher expressivity given by modal operators. We have proposed new neighbourhood semantics for many-valued modal logics. We have also considered a two-layered syntax that turns out to be useful for the management of uncertainty, and we have described the translations between these systems and other logics of uncertainty. We have developed a framework for reasoning with graded predicates; that approach combines Smith’saccount of vagueness as a matter of degree and the algebraic apparatus of mathematical fuzzy logic.

*Highly structured systems.* This area has been in the centre of our research for more than threedecades, in which many remarkable results have been achieved. We have recently studied the role of supermodular functions in game theory and conditional independence structure description. We have also investigated decomposable graphical models, Bayesian networks and other graphical models. A non-graphical approach is, in the Institute, represented by the so-called *compositional models*. We have studied the possibility of using the methods of compositional-model learning to data-mining tasks. We have focused on mining the quantitative and qualitative knowledge that is encoded in compositional models, and which is legible to users from the application fields. We have also contributed to the development of generalized information theories by proposing new ways of defining entropy in the Dempster-

Shafer theory of evidence.

**Applied mathematics**

*Variational analysis and its applications.* We have substantially generalised some standard rules of the generalised differential calculus related to tangent and normal cones to sets havingthe so-called constraint structure and to coderivatives of normal-cone mappings. We have further constructed a new calculus for directional limiting normal cones and subdifferentials, which significantly differs from the classical Mordukhovich calculus. The new rules offer efficient tools for stability analysis of complicated equilibria. Moreover, we have tailored the basic concepts of variational analysis to the quasi-convex setting. We have applied advanced tools of variational analysis to derive optimality conditions and to construct efficient numerical approaches for various application-motivated equilibria.

*Mathematics for mechanics.* Numerous mathematical models in continuum mechanics and thermomechanics of solids described by systems of nonlinear partial differential equations or inequalities have been formulated in a thermodynamically consistent way. They have been mathematically analysed with the aim to prove the existence of their solutions and devise efficient numerical strategies for their approximation implementable on computers. Examples are models in geophysics or in phase transformations in ferromagnetic materials and shape memory alloys. Other instances include plasticity, damage, or adhesive contacts formulated either in small or large strains. Attention has also been paid to various activated, rate-independent/independent processes and to combinations of different phenomena. A new research program has been opened on the justification of linearized models in solid mechanics.

**System science and control theory**

*Large scale systems.* This research direction focuses on a wide range of problems connected with the control of interconnected large-scale systems, synchronisation of multi-agent systems, and complex networks. The stabilisation problem of large-scale systems with time delays and uncertainties has been solved. This is very important since time delays are inevitable in the case of control over communication networks. A novelty not explored so far is the stabilisation of a nonlinear large-scale system with delays in the control loop. Synchronisation of multi-agent systems with delayed controls is another topic that has been explored. Moreover, synchronisation of nonlinear multi-agent systems with and without delays in the control loop has been investigated. The problem of consensus synchronisation has been solved in all cases. Attention has been paid to development of observers for linear as well as nonlinear systems with time delay which will be applied to large-scale and multi-agentsystems in the future.

*Chaotic systems*. The research activity in this area is focused on the important properties ofthe chaotic systems, such as high sensitivity and dependence on the initial conditions, non- periodicity, topological transitivity, etc. This range of problems appears in communication, cryptography, computational neuroscience and other applications. More specifically, the informational signal in the communication systems can be masked by using chaotic systems. Chaos-based random and pseudo-random generators can be used in the cryptography application, computer games or simulations by the Monte Carlo method. There are different types of synchronisation

between coupled chaotic systems like identical synchronisation, generalised synchronisation, phase synchronisation, etc. New methodologies for the detection of generalised synchronisation have recently been proposed.

*Nonlinear predictive control.* The research in this area is concentrated on efficient numerical algorithms in control design where extensive computations are needed. This is the case of predictive control, especially nonlinear model predictive control, spatially distributed systems stemming from the partial differential equations-based models and their applications in iterative learning. The latter, when a linear model is available, can be addressed via multi- variate polynomials requiring, nevertheless, much more demanding and sophisticated computations than classical transfer functions of a single complex variable.

*Decentralised control.* Long term research on decentralised network control allows us to design a control system of earthquake protection devices in high-rise building structures. The resulting control algorithm considerably increases flexibility and reliability; this aspect has been verified by extensive tests. These features are crucial for such applications. Attention hasbeen paid to resistance against failures of some sensors or actuators.

**Probability theory and statistics**

*Stochastic partial differential equations*. Existence of invariant measures for several types of stochastic partial differential equations to which standard methods were not applicable has been established by studying Markov processes having the Feller property with respect to the bounded weak topology of the state space. In this way, existence of an invariant measure for astochastic extensible beam equation has been proved for the first time and results on existenceof invariant measures for stochastic wave equations with polynomial nonlinearities (in both drift and diffusion) have been considerably extended. The developed method has turned out toalso be applicable to (two-dimensional) stochastic Navier-Stokes equations in unbounded domains.

*Interacting particle systems*. The Brownian web is a random continuum object that can loosely be thought of as a collection of coalescing one-dimensional Brownian paths, started from every point in space and time. The Brownian net is a variant of this where paths not onlycoalesce but also branch (separate) with a certain rate. The Brownian web (or net) is known to be the scaling limit of coalescing (resp. coalescing and branching) nearest-neighbour random walks. Since non-crossing properties play an important role in the construction of the Brownian web and net, the convergence is easier to be proved for nearest-neighbour random walks, which share these non-crossing properties. We have studied the convergence of non nearest-neighbour random walks and have achieved original results. We have also studied Recursive Tree Process (RTP), which is a generalisation of a Markov chain where time has a tree-like structure and flows in the direction of the root. More precisely, an RTP consists of a regular rooted tree plus an i.i.d. collection of maps attached to the vertices of the tree, describing the state of that vertex as a function of the states of its descendants.

*Regression methods*. In this field, the research is focused on multiple-output regression methods and statistical survival analysis. We have extended two directional multiple-output quantile regression methods to the locally polynomial regression context. We have implemented the methods and related functions in R and Octave/Matlab in two software packages. We have continued the research of elliptical

multiple-output quantiles and extended them to general location and regression frameworks. In the field of statistical survival analysis, the team has studied the generalisation of random censoring to a dependent competing risk scheme. Namely, conditions of identifiability of the full model (i.e., joint and marginal distributions) have been explored, a rather complicated scheme with regression being considered, where also the corresponding copula parameter could depend on covariates. The problem of stochastic optimisation has been studied in the cases when the stochastic characteristics are estimated from incomplete data, and, in particular, from randomly right- censored data.

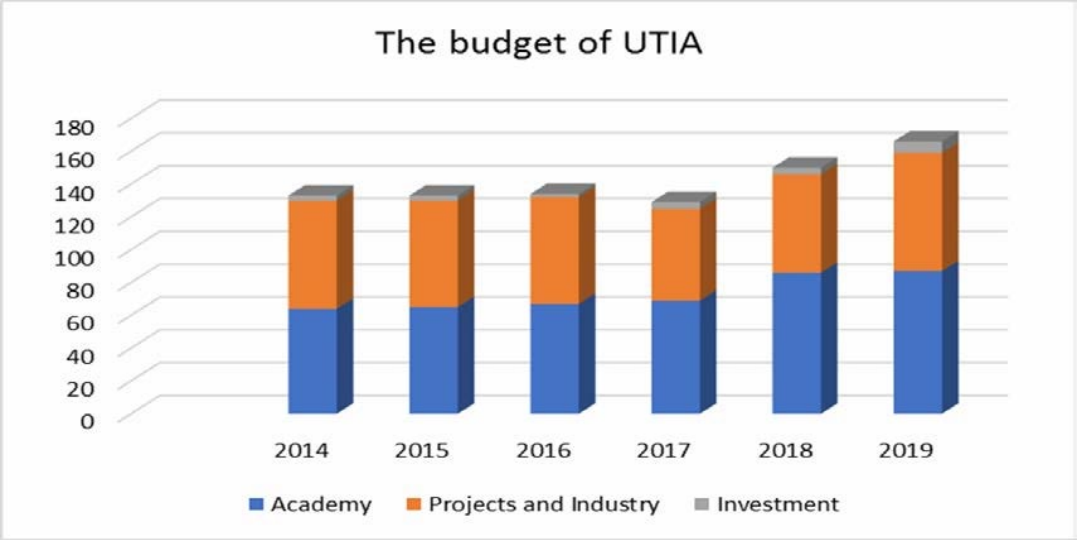
**Research outcomes**

The wide variety of research topics, which range from pure theoretical research to applications, also implies the variety of the outcomes. Since our main focus is on fundamental research, the most important outcomes are research papers. In 2015-2019, our researchers authored/coauthored **10 monographs, 29 book chapters, 405 papers in impacted journals** and several hundreds of others publications, leading to more than **1,000 publications in total**. For the detailed list of publications sorted according to the teams and publication categories, see the materials of individual teams.
A typical outcome of applied research, which is usually funded by application-oriented grant agencies or directly paid by the companies, consists of patents and software solutions.

**Research projects and funding**

The wide portfolio of research directions requires appropriate funding. Since the institutional funding (i.e., the part of the budget provided by the Academy) is less than 50% of the total budget we need, we have been forced to seek an extensive external funding via research grants or industrial contracts.

The 2019 institutional funding amounted to about 78 M CZK, while the total annual budget was about 160 M CZK. The difference is covered by various research projects funded by the EU, the Czech Science Foundation (GACR), the Technology Agency (TACR), the Ministry of Education, and by other public and private sources. A small part of the budget (about 4 M CZK) comes from commercial research. As can be seen from these numbers, ÚTIA is very successful in attracting external funding. In the evaluation period, we were involved in 112 projects and industrial contracts altogether, which brought about 320 M CZK (11.3 M EUR) in 2015-19 (see the graph for individual years).


The budget of UTIA
180
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Projects and Industry
Investment

## Research activity and characterisation of the main scientific results

The achieved research results are reflected in publications and via transfers of theoretical results to diverse applications inspiring us towards further theoretical advances.

The results are clustered according to the main research topics covered by AS department. Every topic may contain several sub-directions. The clusters from the past evaluation are partially preserved in order to simplify the assessment of the research progress and impact.
Samples of the relevant papers are provided. The results referred were achieved by the team members unless otherwise stated. Whenever a result has been achieved in collaboration, the team's share is explicitly described.

### R1: Fully probabilistic design of decision-making strategies

1. Fully probabilistic design (FPD) of decision strategies invented by our team extends theory of dynamic, expected-utility-based, decision making (DM). This has made its consolidation and deepening important [1]. FPD serves us as a research backbone by exploiting the fact that the vast majority of scientific tasks are DM problems. Besides that it has opened an avenue to quantum-probability version of FPD anticipated under the newly established topic B3 of the previous evaluation.
2. The pioneering work on FPD of hierarchical Bayesian models [18] uses FPD to create a stochastic model of the unknown distribution. The approach copes with non-linear functional constraints and equips the distribution estimates with an uncertainty. It provides a novel unified view on FPD-based knowledge merging, adaptive control, approximate learning and tracking. It also has a large impact in knowledge transfer as seen in [11], where a parameter is systematically modified by an external observation-predicting distribution. The gained non-standard transfer of an indirect probabilistic knowledge is optimal in the minimum cross-entropy sense. It allows a randomized, computationally cheap choice of the updated prior with an assessed robustness. It significantly enhances recursive transfer learning and a flat cooperation of adaptive intelligent agents.
3. The following, predominantly conference papers, reflect non-trivial progress in a priori foreseen research directions: [32] concerns FPD based balancing of exploitation and exploration properties of adaptive decision strategies, [34] elaborates FPD towards its real-life applicability by designing universal combinations of probability densities on which FPD operates.

All these results were achieved by the evaluated team members and by their students.

### R2: Decentralised dynamic decision making under uncertainty

A distributed cooperation of imperfect decision makers having multiple DM goals has been further developed. This allowed an *automatic harmonisation* of preferences within a fully scalable flat cooperation scheme. The particular predominantly achieved by our team, results are:

1. Feasibility of the flat FPD-based cooperation concept has been verified by elaborating it to address complexity in distributed control networks. Simple adaptive FPD-based controllers optimise their local control loops. The targeted global behaviour is gained by scalable sharing of probabilistic information among the neighbours. This novel solution is based on adaptively implemented FPD, which copes with uncertainties and ensures systematic information sharing. The complete and tested solution applied to linear-Gaussian local models is [16]. Another test on a linearised coupled map lattice with spatio-temporal chaos provides insight into the obtained emerging behaviour [16]. Team's share: the evaluated team proposed the overall concept and developed theoretical and algorithmic solution.
2. As it was planned, automated DM preferences elicitation respecting agents' fragmental knowledge was addressed. A novel general principle of preference elicitation was proposed [31] and successfully applied to a standard linear quadratic control problem. The principle can be seen as a preference-oriented counterpart of minimum relative entropy principle serving to knowledge elicitation. Its use will bring a range preference elicitation algorithms serving to various DM scenarios.
4. The progress in merging probabilities FPD operates on [34] enabled cooperation of independent selfish agents with partially contradictory aims [36]. It belongs to the newly established research direction that respects deliberation effort of DM agents [35] or their emotional state [33]. This has opened a way towards implicit cooperation [37] and a deeper elaboration of human-like aspects of DM.

**R3: Advances in Bayesian estimation**

The traditional AS domain brought a range of results, predominantly created by members of the evaluated team. The reported achievements support the significance of the originally planned research:

1. A novel approximate Bayesian computation (ABC) filtering paradigm, for sequential estimation with inexact likelihood deals with nonlinear state-space models with a linear substructure estimated via Kalman-type filter. It is the first ABC application to marginalised filtering, which provides a new robust method for evaluation of particle weights [7]. Its robustness is enhanced by a computationally cheap specification of kernel bandwidth [14].
2. Approximate recursive Bayesian estimation of models with bounded noise helps whenever there are hard bounds on the modelled quantities, like in robotics. This makes the novel solution evolving parallelotopes even without knowledge of noise level so important [12].
3. A systematic analysis led to estimator of a high-order Markov chain that universally model dynamic relations of discrete data. The solution: i) overcomes the inherent curse of dimensionality; ii) complements its systematic approximate Bayesian on-line estimator by a universal, data-dependent, stabilised forgetting that prevents accumulating approximation errors [25].
4. Diffusion estimation allows for collaborative inference of parameter of interest using a spatially distributed network, where each node can exchange information with its adjacent neighbours. Unlike the existing single-problem-

oriented solutions our proposed framework is universally applicable to a wide class of diffusion networks [9]. The work [13] has been prepared in collaboration. Team's share: the evaluated team contributed with the initial idea and theory of collaborative estimation and implemented simulations.

**R4: Specialised tasks in applied mathematics**

The general orientation of the evaluated team can be perceived as an applied mathematics. Some solutions motivated by specific purposes have a broader use and thus it makes sense to present them under a separate cluster.

1. The need to get non-negative matrix factorisation motivated the reported research. It was prompted by an application in dynamic image sequences (for instance functional magnetic resonance imaging or dynamic renal scintigraphy) but has far-reaching application potential. The offered solution [3],[23] systematically copes with sparsity and smoothness without using ad hoc weights and outperforms the state-of-the-art methods.
2. At present, nobody has doubts about the importance of viral infection models. The research oriented on dynamic systems with diffusion and state-dependent delays [27],[28] (team's share: the evaluated team contributed with overall idea, theoretical development and proofs) offered such a model for first time [6[,[29].
3. Duality-based nonlinear quadratic control [30] is another typical example where the theoretical solution applied to a specific case has much broader potential impact. This one finds a significantly faster solution than the existing iterative linear-quadratic-regulation on the problem of robot trajectory following. Team's share: the evaluated team contributed with formulation of duality between extended Rauch-Tung-Striebel smoother and non-linear quadratic control, derived and implemented the main algorithm.
4. A significant progress has been achieved on analysis of parabolic partial differential equations with discrete state-dependent delay [21] (team's share: the evaluated team contributed with analysis and proving the main theorems). The works of this type prepare applicable algorithms as that in [24] (team's share: the evaluated team prepared major part of theoretical results, supervised numerical computations and helped to evaluate its results and participated in final results processing)

**R5: Complex modelling for decision support**

This research direction is strongly application-motivated but brought important, broadly applicable theoretical conclusions. The international project on a source-term determination of radionuclide releases by inverse atmospheric dispersion modelling was the main driving force of this research.

1. Data from gamma dose rate detectors are the most accessible measurements of atmospheric release but they provide data that comprise contributions of all nuclides from both plume and deposition. The corresponding quite difficult source determination was addressed within Bayesian paradigm while exploiting a prior knowledge on nuclide ratios given in the form of bounds. This enables analysis of emergency radiation situations as well as of other atmospheric applications [4] (team's share: the evaluated team contributed to experiment design, prepared and performed main experiments). Variational Bayes model selection with a novel adaptive choice of prior covariance was

developed. Its significant application potential for source location of unknown atmospheric pollutant release was confirmed. Ambient concentration measurements of iodine-131 released during an incident in 2011 allowed us to find the release location, its magnitude and temporal variation. Our results were confirmed by an independently reported information about the release [8] (team's share: the evaluated team designed and performed the experiments, supervised the study).

2. The proposed recursive parameter estimation and tracking of the radioactive plume propagation allows fast estimating model parameters in the complex multi-segment and multi-nuclide scenarios, which is vital for real-time evaluation. The proposed approach has a high application potential in efficient nuclear emergency management [20]. Estimating pollutant release is typically formulated as a linear inverse problem. Its results heavily depend on a range of optional parameters that motivated the proposed data-based tuning. The method is validated on data from the European Tracer Experiment and its stability was demonstrated [17] (team's share: the evaluated team contributed with experiment design, prepared and performed main experiments).
3. Image restoration problem is quite a common subtask often deciding on DM quality. This particular problem was solved within Bayesian framework. The proposed prior [5] (team's share: the evaluated team proposed the prior model and designed the identification algorithm) has a rich structure that allows fitting closely the restored image yet it is possible to estimate its hyper-parameters analytically. The proposed algorithm outperforms other prior models in certain regimes of blind super-resolution task. The main impact of this prior model is that the estimated prior image closely resembles the restored image. This is a unique property that is not common for alternative prior models.
4. A similar methodological contribution, motivated by an important ecological problem, is solved in [26] (team's share: the evaluated team contributed with formula derivations for exponential and Pareto case, formula derivations for saddle point approximations, relation of I-divergence to other divergences in information theory). The article [26] is an excellent example of multidisciplinary collaboration of theoretical and applied fields, such as statistics, information theory, ecology and computer programming. The contribution lies in theoretical development and demonstration of a graphical tool for chemometricians to check for fit of assumed models and to evaluate the amount of chaos in data. The tool is applicable to a variety of ecological data and offers help with data assessment in other applied fields.

**R6: Advances in nonlinear, and decentralized control**

The achieved theoretic progress within this matured, permanently-developing domain concerns the following results:

1. Multi-step ahead Lyapunov function approach used in analysis and synthesis of switched systems has improved ability to attenuate disturbances while relaxing average dwell time constraint [2] (team's share: the evaluated team contributed to the article concept, theory development, programming). This has a notable application potential.
2. A novel predictive control of speed and position of 3-phase permanent magnet synchronous motors is described in [22] (team's share: the evaluated team

proposed the novel explicit predictive methods including fast procedure for solving the constraints on currents). The proposed controller copes with step and ramp reference signals and respects current limitation using a local indirect tuning of the control parameters. It was verified on the 10kW drive and is ready for practical use.

3. Long term research on decentralised network control allowed designing control system of the earthquake protection devices in the high rise building structures. The resulting control algorithm considerably increases flexibility and reliability as was verified by the extensive tests. These features are crucial for such applications. Attention was paid to resistance against failures of some sensors or actuators [19] (team's share: the evaluated team prepared major part of theoretical results, supervised numerical computations and performed final results processing).
4. Delays are known to be critical for stability of large scale and complex controlled systems. An original method stabilizing the symmetrically interconnected large-scale system with time delays was proposed in [24] (team's share: the evaluated team prepared major part of theoretical results, supervised numerical computations and helped to evaluate its results and participated in final results processing). Importantly, the delays can differ in each subsystem which is vital for controlling practical large-scale systems over communication networks. Also, the rapid changes of the time delays can be handled.

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## Research activity and characterisation of the main scientific results

Structured by the above characterization of the team focus, the description of the research activity of the **Department of Control Theory (DCT**) during 2015-2019 is as follows. For better orientation the research activities are loosely divided into the following main directions, though these directions are closely related and some results thereby may contribute to more than one of these directions.

### 1. Modelling and control of mechanical systems with applications to robotic walking

This research direction has ranged from theoretical development based on classical mechanics systems modelling and control up to experimental laboratory research based on building walking like platform consisting of two legs with torso actuated at ankles and hips. Control design was based on the notion of the so-called virtual holonomic constraints and input-output exact feedback linearization. The new and more specific types of constraints were introduced, the collocated virtual holonomic constraints. Furthermore, the methods for the estimation and identifications of the underactuated walking robots were studied. This overall research direction is focused on the underactuated walking where the pivot angle at the contact point of the stance leg is not directly actuated, moreover, this angle is also only indirectly measured. Main focus was to the simplified two-dimensional models addressing only forward dynamics in sagittal plane. Therefore, the laboratory model requires some laterally supportive construction. Nevertheless, the natural lateral stability for the future possibly three-dimensional walking models was studied as well. The latter may provide some simpler alternatives to the study of overly complex three-dimensional mechanical models. The specific results achieved within this area are:

**Collocated virtual holonomic constraints in 2D walking design.** Theory of the collocated virtual holonomic constraints and their enforcement based on Lagrangian and Hamiltonian representations in the special generalized coordinates was developed and investigated. These special coordinates are actually such that constraints are linear (aka flat) subspaces of the configuration space. Design of the collocated virtual holonomic constraints defining suitable walking-like patterns and their use for the computation of the walking feedback controllers, including their stability design and analysis. Simulation verification was also performed. The result was achieved by the evaluated team members only and its details were published in the following papers:

***S. Čelikovský.*** Flatness and realization of virtual holonomic constraints in Lagrangian systems, IFAC-PapersOnLine. Volume 48, Issue 13, p. 25-30, The 5th IFAC Workshop on Lagrangian and Hamiltonian Methods for Non-Linear Control (2015), Lyon, FR.
***S. Čelikovský, M. Anderle.*** On the collocated virtual holonomic constraints in Lagrangian systems, Proceedings of the American Control Conference (ACC), 2016, p. 6030-6035, The American Control Conference (ACC) 2016, Boston, US.
***M. Anderle, S. Čelikovský.*** On the hybrid stability of the collocated virtual holonomic constraints-based walking design, Cybernetics and Physics, Vol. 6 (2017), pp. 47-56
***S. Čelikovský, M. Anderle.*** Hybrid invariance of the collocated virtual holonomic constraints and its application in underactuated walking. In Preprints of the 10th IFAC Symposium on Nonlinear Control Systems, pages 802-807, Monterey, CA, USA, 2016.

***S. Čelikovský, M. Anderle.*** Collocated Virtual Holonomic Constraints in Hamiltonian Formalism and Their Application in the Underactuated Walking. In Proceedings of the 11th Asian Control Conference (ASCC) 2017, pp. 192-197.
***S. Čelikovský, M. Anderle.*** On the Hamiltonian approach to the collocated virtual holonomic constraints in the underactuated mechanical systems. In Lecture Notes in Electrical Engineering - 4th International Conference on Advanced Engineering Theory and Applications, AETA 2017, vol. 465, pages 554-568, Ho Chi Minh; Viet Nam, 2018.
***S. Čelikovský, M. Anderle.*** Stable walking gaits for a three-link planar biped robot with two actuators based on the collocated virtual holonomic constraints and the cyclic unactuated variable, IFAC PapersOnLine. Vol. 51, Issue 22: 12th IFAC Symposium on Robot Control SYROCO 2018, pp. 378-385.
***S. Čelikovský, M. Anderle.*** Exact feedback linearization of the collocated constrained dynamics of the three-link with adjustable torso and its application in the underactuated planar walking. In Proceedings of the 15th IEEE International Conference on Control and Automation (ICCA), pp. 1289-1295, Edinburgh, Scotland, 2019.

**Laboratory experiments related results: identification with undirect and noisy unactuated angle measurements, sensor fusion and real movement control.** Besides results published in scientific articles, this direction included extensive mechanical and electronical hardware investigation for the laboratory model construction and improvement. Significant progress when solving the problem of parameters estimation and model identification for a class of underactuated mechanical systems modelled via the Euler–Lagrange formalism was achieved. Most importantly, noisy and nonlinearly deformed unactuated absolute orientation angle measurement was handled using the original nonlinear optimization method. The method was successfully tested both in simulations and by experiments using real laboratory underactuated walking model, for the feedback control purposes, the sensor fusion methods were studied to improve on-line measurements using their redundancy. Special attention was devoted to preparing the so-called low-level implementation using simple arithmetic only, the outlook is to perform Kalman filter based sensor fusion using simple processors on PC boards attached to legs only, avoiding central computer processing. This result contributed to the list 15 selected results of the evaluated DCT team (Part 3.3 – different file of evaluation materials delivered earlier) by one publication published in Q1 journal. The result was achieved by the evaluated team members only and its details were published in the following papers:

***M. Anderle, S. Čelikovský and K. Dolinský*****.** Simple model of underactuated walking robot, Proceedings of the 10th Asian Control Conference (ASCC 2015). ISBN 978-1-4799-7862-5, Kota Kinabalu, MY.
***M. Anderle, S. Čelikovský.*** On the sensor fusion in the walking robots design, Proceedings of the 11th Asian Control Conference (ASCC) 2017, p. 2534-2539, The 11th Asian Control Conference (ASCC) 2017.
***M. Anderle, S. Čelikovský.*** Sensor Fusion for simple walking robot using low-level implementation of Extended Kalman Filter, IFAC-PapersOnLine. Vol. 51, Is. 13.: 2nd IFAC MICNON 2018 (Conference on Modelling, Identification and Control of Nonlinear Systems), pp. 43-48.
***K. Dolinský, S. Čelikovský.*** Application of the Method of Maximum Likelihood to Identification of Bipedal Walking Robots, IEEE Transactions on Control Systems

Technology vol. 26 (2018), pp. 1500-1507. ***Included in the DCT team 15 selected results.***
***M. Anderle, S. Čelikovský.*** On the controller implementation in the real underactuated walking robot model, Proceedings of the 12th Asian Control Conference (ASCC 2019), pp. 1125-1130, Asian Control Conference (ASCC 2019), Kitakyushu International Conference Center, JP.

**Natural lateral stability of the three-dimensional walking**. The novel original idea to study lateral dynamics of the possibly 3D walking-like systems to simplify otherwise overly complex 3D design was introduced. Lateral dynamics is represented by the benchmark model - hybrid inverted pendulum – whose periodic motions stability is analysed and shown to be preserved under limited lateral periodic forcing. Moreover, the respective bounded behavior may be even chaotic and a very rich variety of attractors was demonstrated. This result contributed to the list 15 selected results of the evaluated DCT team (Part 3.3 – different file of evaluation materials delivered earlier) by one publication. The result was achieved by the evaluated team members only, it contributed to and its details were published in the following papers:

***S. Čelikovský, V. Lynnyk.*** On the chaotic behavior of the hybrid inverted pendulum and its relation to the lateral dynamics of the walking robots, IFAC-PapersOnLine. Vol. 51, Is. 33: 5th IFAC Conference on Analysis and Control of Chaotic Systems CHAOS 2018, Eindhoven, NL, pp. 15-21.
***S. Čelikovský, V. Lynnyk.*** Lateral Dynamics of Walking-Like Mechanical Systems and Their Chaotic Behavior, International Journal of Bifurcation and Chaos, vol.29 (2019), pp. 1930024-1 - 1930024-29. ***Included in the DCT team 15 selected results.***

**Related problems in mechanical systems control and the flight dynamics.** Here, the sub-optimal control solution to magnetic manipulation problem was obtained, including its experimental implementation. Moreover, benchmark problem in flexible systems was studied, namely, the pendulum with variable length and its damping. Finally, the design of a longitudinal flight controller for a fixed-wing aircraft using non-linear dynamic inversion technique or, in terms of control theory, partial exact feedback linearization. The internal dynamics have been shown to be exponentially stable. This makes the resulting controllers suitable for various flight conditions. This result contributed to the list 15 selected results of the evaluated DCT team (Part 3.3 – different file of evaluation materials delivered earlier) by one publication. This area was developed in close collaboration with the Czech Technical University in Prague (Faculty of Mechanical Engineering and Faculty of Electrical Engineering) and University of Leuven, see shares bellow. Where not stated otherwise, the share of DCT team is 100%. The specific details were published in the following publications:

***M. Alam M, S. Čelikovský. D. Walker:*** Robust Hover Mode Control of a Tiltrotor Using Nonlinear Control Technique, AIAA Guidance, Navigation, and Control Conference, pp. 1-16, The AIAA Conference on Guidance, Navigation, and Control, (2016), (San Diego, California, US. **DCT team members research share estimate is 20%.**
***M. Alam M, S. Čelikovský.*** On the internal stability of non-linear dynamic inversion: application to flight control, IET Control Theory and Applications, vol.11, 12 (2017), pp. 1849-1861. **DCT team members research share estimate is 50%.** ***Included in the DCT team 15 selected results.***

***J. Zemánek, S. Čelikovský, Z. Hurák:*** Time-optimal Control for Bilinear Nonnegative-in-control-Systems: Application to Magnetic Manipulation, IFAC-PapersOnLine. Volume 50, Issue 1: 20th IFAC World Congress, p. 16032-16039. ***Awarded*** *the* ***EEA Demonstrator Paper Prize*** *at the IFAC World Congress 2017 in Toulouse. This special demonstrator prize is funded by Club EEA - French association of professors and researchers in electrical and information sciences.* **DCT team members research share estimate is 40%.**
***M. Anderle, W. Michiels, S. Čelikovský, T. Vyhlídal:*** Damping a pendulum's swing by string length adjustment - design and comparison of various control methods, Proceedings of the 2019 American Control Conference (ACC), p. 4399-4405. **DCT team members research share estimate is 60%.**

**2. Large scale systems, their control and observation under time delays. Complex networks, networked dynamical systems and their control based on invariant manifolds.**

This research direction focuses on a wide range of problems connected with control of interconnected large-scale systems, synchronization of multi-agent systems and complex networks. Stabilization problem of large-scale systems with time delays and uncertainties was solved. This is very important as time delays are inevitable in the case of control over communication networks. A novelty not explored so far was stabilization of a nonlinear large-scale system with delays in the control loop. Synchronization of multi-agent systems with delayed controls was another topic to be explored. In particular, multi-agent systems with heterogeneous delays (different delays in various agents) were studied. This problem poses some challenges over the setting when the delays of all agents are identical as variability of delays introduces a kind of non-symmetry. Moreover, synchronization of nonlinear multi-agent systems with and without delays in the control loop was investigated. The problem of consensus synchronization was solved in all cases. Attention was paid to development of observers for linear as well as nonlinear systems with time delay which will be applied to large-scale and multi-agent systems in future. The following specific results were achieved:

**Stabilization of large-scale systems.** Algorithms for stabilization of large-scale interconnected systems controlled by control networks were proposed. They were designed using the robust control theory, the main tool are linear matrix inequalities so that resulting algorithms can be easily implemented. Attention is also devoted to robustness against failures of some of the controllers. The first result was obtained for systems with symmetrical interconnections (every subsystem is connected with all remaining subsystems). Then, a generalization to a system with arbitrary interconnections was proposed. Moreover, the latter result is derived using the Wirtinger inequality, hence achieving better disturbance attenuation. Nonlinear large-scale systems were also studied. Here, the exact feedback linearization was used first, then the algorithm for stabilization of linear systems was used. Effects of quantization in the case of large-scale control as well as control of discrete-time large-scale systems were investigated. Application of the decentralized control to the system for a protection of a building against earthquake damage was developed. This result contributed to the list 15 selected results of the evaluated DCT team (Part 3.3 – different file of evaluation materials delivered earlier) by three publications. Its smaller part was developed in collaboration with other teams of ÚTIA, see shares bellow.

Where not stated otherwise, the share of DCT team is 100%. The result was achieved by the evaluated team members only and its details were published in the following papers:

***B. Rehák.*** Large-Scale Systems Control Design via LMI Optimization, Information Technology and Control, vol.44, 3 (2015), p. 247-253.
***L. Bakule, B. Rehák, M. Papík*****.** Decentralized H-infinity control of complex systems with delayed feedback, Automatica vol.67, 1 (2016), pp. 127-131. **DCT team members research share estimate is 40%.** ***Included in the DCT team 15 selected results.***
***L. Bakule, B. Rehák, M. Papík*****.** Networked Control of Building Structures, Computer-Aided Civil and Infrastructure Engineering vol.31, 11 (2016), pp. 871-886. **DCT team members research share estimate is 40%.** ***Included in the DCT team 15 selected results.***
***B. Rehák, V. Lynnyk.*** Network-based control of nonlinear large-scale systems composed of identical subsystems, Journal of the Franklin Institute-Engineering and Applied Mathematics vol.356, 2 (2019), pp. 1088-1112. ***Included in the DCT team 15 selected results.***
***B. Rehák, V. Lynnyk.*** Decentralized networked stabilization of a nonlinear large system under quantization, IFAC-PapersOnLine. Volume 52, Issue 20 - Proceedings of the 8th IFAC Workshop on Distributed Estimation and Control in Networked Systems, Chicago, IL, USA 2019, pp. 49-54.
***B. Rehák, V. Lynnyk.*** Robust stabilization of a discrete-time large-scale interconnected system composed of identical subsystems, Proceedings of the 24th International Conference on Methods and Models in Automation and Robotics (MMAR 2019), pp. 179-184.
***B. Rehák.*** Wirtinger inequality-based control design for an interconnected large-scale system with sampled controls, Proceedings of the 38th Chinese Control Conference 2019, pp. 1009-1014.

**Synchronization of multi-agent systems and complex networks.** In the rapidly developing field of multi-agent systems, synchronization problem for a multi-agent system composed of identical agents was solved. The most distinctive feature is the use of the exact feedback linearization in the design process. This procedure is applied first, then synchronization scheme for linear multi-agent systems is applied. Moreover, a nontrivial zero dynamics can be present, however, it is supposed the agents are minimum-phase systems. The case without delays was investigated first, then, in the complex network setting, synchronization of a nonlinear multi-agent system with delayed controls was studied. As an important result, it was shown that in the delay-free case, synchronization of the agents can be achieved, however, delays can cause some synchronization error whose magnitude does not converge to zero. Estimates of this error were proposed. Also, effects of heterogeneous time delays in a complex network with linear nodes were investigated. It was shown again that, in general, convergence of the synchronization error to zero cannot be achieved, however, the limit of the synchronization error at infinity can be estimated. Last but not least, the attention was paid to investigation of the fixed consensus combined with switching interconnections. This result contributed to the list 15 selected results of the evaluated DCT team (Part 3.3 – different file of evaluation materials delivered earlier) by two publications. This result was achieved partly by the collaboration of the evaluated team members with Faculty of Electrical Engineering, Czech Technical University in Prague

and CINVESTAV del I.P.N, Mexico, see shares bellow. Where not stated otherwise, the share of DCT team is 100%. Its details were published in the following papers:

***R. Casas-Carrillo, O. Begovich, J. Ruiz-León, S. Čelikovský***. Adaptive Fault Diagnoser based on PSO Algorithm for a class of Timed Continuous Petri Nets, Proceedings of 2016 IEEE 21th Conference on Emerging Technologies & Factory Automation (ETFA), pp. 1-7, Berlin, Germany. **DCT team members research share estimate is 25%.**

***K. Hengster-Movric, M. Šebek, S. Čelikovský.*** Structured Lyapunov functions for synchronization of identical affine-in-control agents-Unified approach, Journal of the Franklin Institute-Engineering and Applied Mathematics, vol.353, 14 (2016), pp. 3457-3486. **DCT team members research share estimate is 33%. *Included in the DCT team 15 selected results.***

***B. Rehák, V. Lynnyk, S. Čelikovský.*** Consensus of homogeneous nonlinear minimum-phase multi-agent systems, IFAC-PapersOnLine. Vol. 51, Is. 13: 2nd IFAC Conference on Modelling, Identification and Control of Nonlinear Systems MICNON 2018, pp. 223-228, Guadalajara, MX, 20180620.

***D. Goméz-Gutiérrez, J. Ruiz-León, S. Čelikovský, J.D. Sánchez-Torres***. A Finite-Time Consensus Algorithm with Simple Structure for Fixed Networks, Computación y Sistemas, vol.22, 2 (2018), pp. 547-556. **DCT team members research share estimate is 25%.**

***B. Rehák, V. Lynnyk.*** Synchronization of symmetric complex networks with heterogenous time delays, Proceedings of the 22nd International Conference on Process Control (2019), pp. 68-73, Eds: Kvasnica M., Štrbské Pleso, SK, 20190611.

***B. Rehák, V. Lynnyk.*** Synchronization of nonlinear complex networks with input delays and minimum-phase zero dynamics, Proceedings of the 19th International Conference on Control, Automation and Systems (ICCAS 2019), pp. 759-764, Jeju, KR.

***D. Gómez-Gutiérrez, C.R. Vázquez, S. Čelikovský, J.D. Sánchez-Torres J, J. Ruiz-León.*** On finite-time and fixed-time consensus algorithms for dynamic networks switching among disconnected digraphs, International Journal of Control, DOI: 10.1080/00207179.2018.1543896. **DCT team members research share estimate is 20%. *Included in the DCT team 15 selected results.***

**Auxiliary results concerning observers with applications.** Observers are necessary for practical implementation of control, algorithms, they will be used in the aforementioned problems as well. Hence the need for developing a reliable observer for linear as well as nonlinear systems, especially in presence of time delays. One way of construction of nonlinear observers is based on the solution of a certain partial differential equation. This solution is hard to find however the finite element method was successfully applied to overcome this issue. This approach can be adopted even for design of observers for systems with delayed measurements provided the time delay is constant and known. Application of this method in the delay-free case to a biological system was presented. Besides, for a special case of polynomial systems, an observer design based on the sum-of-squares method was proposed. Moreover, an observer for a linear time-delayed system with quantization was proposed, this design is based on linear matrix inequalities. An estimate of the observation errors arising when the time delays of the observed system and the observer differ was derived. Observers for switched linear systems were investigated as well which have application potential for the chaotic encryption as switched linear systems can be used

to generate chaotic behaviour. This result contributed to the list 15 selected results of the evaluated DCT team (Part 3.3 – different file of evaluation materials delivered earlier) by two publications. The result was partly achieved via cooperation of the evaluated team members with CINVESTAV del I.P.N., Mexico, see shares bellow. Where not stated otherwise, the share of DCT team is 100%. Its details were published in the following papers:

***D. Gómez–Gutiérrez, S. Čelikovský, A. Ramírez–Trevino, B. Castillo-Toledo.*** On the observer design problem for continuous-time switched linear systems with unknown switchings, Journal of the Franklin Institute-Engineering and Applied Mathematics, vol. 352, 4 (2015), pp. 1595-1612. **DCT team members research share estimate is 25%.** ***Included in the DCT team 15 selected results.***
***B. Rehák.*** Sum-of-squares based observer design for polynomial systems with a known fixed time delay, Kybernetika, vol.51, 5 (2015), pp. 858-873.
***B. Rehák.*** Observer Design for a Time Delay System via the Razumikhin Approach, Asian Journal of Control vol.19, 6 (2017), pp. 2226-2231. ***Included in the DCT team 15 selected results.***
***D. Gómez-Gutiérrez, C. Renato Vázquez, S. Čelikovský, A. Ramírez-Trevino, B. Castillo-Toledo.*** On the distinguishability and observer design for single-input single-output continuous-time switched affine systems under bounded disturbances with application to chaos-based modulation, European Journal of Control vol. 34, 1 (2017), pp. 49-58. **DCT team members research share estimate is 20%.**
***S. Čelikovský, J. A. Torres-Munoz, A. R. Dominguez-Bocanegra.*** Adaptive high-gain observer extension and its application to bioprocess monitoring, Kybernetika, vol.54, 1 (2018), pp. 155-174. **DCT team members research share estimate is 70%.**
***B. Rehák.*** Finite-element based observer design for nonlinear systems with delayed measurements, IFAC-PapersOnLine. Vol. 51, Is. 14.: 14th IFAC Workshop on Time Delay Systems TDS 2018, pp. 201-206. Budapest, Pesti Vigado, HU.
***B. Rehák.*** Finite element-based observer design for nonlinear systems with delayed measurements, Kybernetika, vol.55, 6 (2019), pp. 1050-1069.
***B. Rehák, V. Lynnyk.*** Design of a nonlinear observer using the finite element method with application to a biological system, Cybernetics and Physics, vol.8, 4 (2019), pp. 292-297.

## 3. General systems theory, chaotic systems synchronization and its applications.

The research activity in this area focused the potentially useful properties of the chaotic systems like high sensitivity dependence on the initial conditions, non-periodicity, topological transitivity, etc., in the different applications like communication, cryptography, computational neuroscience and other. More specifically, the informational signal in the communication systems can be masked by using chaotic systems. Chaos-based random and pseudo-random generators can be used in the cryptography application, computer games or simulations used the Monte Carlo method. One of the simplest, by very important futures of the collective dynamics, is synchronization between interconnected systems, including nonlinear and chaotic systems. There are different types of synchronization between coupled chaotic systems like identical synchronization, generalized synchronization, phase synchronization, etc. Chaotic systems can be interconnected in a different topology, like star, ring, complete, etc. The detection of the identical synchronization in networks

consisting of chaotic systems with different topologies is a quite trivial task, but the detection of the generalized synchronization between coupled systems with ring topology is not. New methodologies for the detection of generalized synchronization has been proposed recently. The specific results achieved within this area are:

**Application of the chaos theory in the secure encryption.** The theory of the different types of observers is investigated and used for the application of chaos theory in communication and cryptography. Design of the novel chaotic masking scheme via message embedded synchronization was developed. Using the chaotic generalized Lorenz system (GLS) for the masking of the informational signal or generation of the pseudo-random number was investigated. A general class of the systems allowing the message embedded synchronization was studied, moreover, it was shown that the GLS belongs to this class. New algorithms were introduced for the binary sequence generation based on the GLS. Basic statistical tests and security analysis of the pseudo-random number generators (PRNG) based on the GLS are also provided. It was shown that PRNG based on GLS can be a good candidate for using for application in cryptography. The result was partly achieved via cooperation of the evaluated team members with CINVESTAV del I.P.N., Mexico and Nanzan University, Nagoya, Japan, see shares bellow. Where not stated otherwise, the share of DCT team is 100%. Its details were published in the following papers:

***D. Gómez-Gutiérrez, C. Renato Vázquez, S. Čelikovský, A. Ramírez-Trevino, B. Castillo-Toledo.*** On the distinguishability and observer design for single-input single-output continuous-time switched affine systems under bounded disturbances with application to chaos-based modulation, European Journal of Control vol. 34, 1 (2017), pp. 49-58. **DCT team members research share estimate is 20%.**
***S. Čelikovský, V. Lynnyk.*** Message Embedded Chaotic Masking Synchronization Scheme Based on the Generalized Lorenz System and Its Security Analysis. International Journal of Bifurcation and Chaos vol.26, 8 (2016), pp. 1650140-1 – 1650140-15. ***Included in the DCT team 15 selected results.***
***V. Lynnyk, N. Sakamoto, S. Čelikovský.*** Pseudo random number generator based on the generalized Lorenz chaotic system. IFAC-PapersOnLine. Volume 48, Issue 18, p. 257-261. The 4th IFAC Conference on Analysis and Control of Chaotic Systems, (2015), Tokyo, JP. **DCT team members research share estimate is 80%.**
***M. García-Martínez, L. J. Ontanon-García, E. Campos-Cantón., S. Čelikovský.*** Hyperchaotic encryption based on multi-scroll piecewise linear Systems, Applied Mathematics and Computation, vol. 270, 1 (2015), pp. 413-424.

**Generalized synchronization in the interconnected chaotic systems.** One of the important kinds of synchronization is the generalized synchronization (GS) which implies a synchronization between non-identical systems or identical systems with different parameters. The main synchronization condition in GS is the existence of the functional relationship between interconnected systems. The so-called duplicated systems approach for the detection of the GS in a complex network (CN) with ring topology and bidirectional coupling was proposed. Another method of the detection of generalized synchronization is the auxiliary system approach. This method is based on the comparison of the synchronization error between permanent and auxiliary systems involving the same driving signal. The study of the synchronization of the two interconnected chaotic systems logically has been expanded to the research of synchronization in complex networks consist of chaotic oscillators. It will be shown

analytically and by numerical simulations that these methods can be used for the detection of GS between chaotic systems connected in a closed chain. The result was achieved fully by members of the evaluated DCT team. Its details were published as follows:

***V. Lynnyk, B. Rehák, S. Čelikovský.*** On detection of generalized synchronization in the complex network with ring topology via the duplicated systems approach, Proceedings of the 8th International Conference on Systems and Control, pp. 251-256, Marrakech, MA, 2019.
***V. Lynnyk, B. Rehák, S. Čelikovský.*** On applicability of auxiliary system approach in complex network with ring topology, Cybernetics and Physics, vol. 8, 3 (2019), pp. 143-152.

**General systems theory and methods to study chaotic systems**. Here, fundamental and more abstract issues related to complex and chaotic systems were studied. The representation of general dynamical systems by the so-called flows were introduced and studied. This problem is related to solving certain functional equation. Furthermore, the application of the analytical series technique to study properties of the nonlinear chaotic dynamical systems was presented. More specifically, Laplace–Adomian decomposition method is applied to Roessler system and the so-called generalized Lorenz system. The result was partly achieved via cooperation of the evaluated DCT team members with retired expert L. Klapka. Where not stated otherwise, the share of DCT team is 100%. Details were published as follows:

***P. Augustová, L. Klapka***. General solution of Chládek's functional equations, Journal of Difference Equations and Applications vol.21, 10 (2015), pp. 997-1001.
**DCT team members research share estimate is 70%.**
***Z. Beran, S. Čelikovský.*** Numerical analysis of the Adomian method and its application to chaotic systems. IFAC-PapersOnLine. Volume 49, Issue 18, p. 368-373, The 10th IFAC Symposium on Nonlinear Control Systems, Monterey, California, USA (2016), Monterey, California, US.
***Z. Beran, S. Čelikovský.*** Analytical-Algebraic Approach to Solving Chaotic System. International Journal of Bifurcation and Chaos, vol.26 (2016), pp. 1650051-1 - 1650051-13. ***Included in the DCT team 15 selected results.***

## 4. Numerical algorithms for predictive control, distribute systems and iterative learning.

The research in this area was concentrated on efficient numerical algorithms in control design where extensive computations are needed. This is the case of predictive control, especially nonlinear model predictive control, spatially distributed systems stemming from the partial differential equations-based models and their application in iterative learning. The latter, when linear model is available, can be addressed via multi-variate polynomials requiring, nevertheless, much more demanding and sophisticated computations than classical transfer functions of single complex variable. The following specific result were achieved.

**Predictive optimization with applications.** Computationally tractable algorithm focusing on overall optimization of a production processes is developed. It has three principal stages: (i) optimization of the input profile with fixed initial conditions, (ii)

reduction of the input profile complexity and (iii) joint optimization of the input profile parameters and state initial conditions. The algorithm is successfully compared with several alternatives on a series of numerical experiments representing penicillin cultivation process. Furthermore, the gap between the linear (simplified convex setting) and the nonlinear (more precise, but complex computations) model predictive control is bridged by introducing a predictive controller with linear time-dependent model. Finally, Hamiltonian-switching hybrid nonlinear predictive control algorithm for car motion predictive control is developed. It incorporates the information about hybridity directly into the optimization routine thereby avoiding computationally demanding nonlinear mixed-integer programming task. All developed alternatives are verified on an example of a motion control of a racing car and compared with the approximation-based nonlinear predictive control and a commercial product. The result contributed to the list of 15 selected team publications by three items and it has been partly achieved via cooperation of the evaluated team members with Faculty of Electrical Engineering, Czech Technical University in Prague, see shares bellow. Its details were published in the following papers:

***M. Pčolka, S, Čelikovský.*** Production-process optimization algorithm: Application to fed-batch bioprocess, Journal of the Franklin Institute-Engineering and Applied Mathematics, vol.354, 18 (2017), pp. 8529-8551. **DCT team members research share estimate is 40%.** ***Included in the DCT team 15 selected results.***
***M. Pčolka, E. Žáčeková, Robinett, S. Čelikovský, M. Šebek.*** Bridging the gap between the linear and nonlinear predictive control: Adaptations fo refficient building climate control, Control Engineering Practice, vol.53, 1 (2016), pp. 124-138. **DCT team members research share estimate is 10%.** ***Included in the DCT team 15 selected results.***
***M. Pčolka, E. Žáčeková, Robinett, S. Čelikovský, M. Šebek.*** Toward a Smart Car: Hybrid Nonlinear Predictive Controller with Adaptive Horizon IEEE Transactions on Control Systems Technology, vol.26, 6 (2018), pp. 1970-1981. **DCT team members research share estimate is 10%.** ***Included in the DCT team 15 selected results.***

**Fast numerical test of multivariate polynomial positiveness with applications**. It is well known that spatially distributed systems can be described by multivariate polynomial fractions. The stability of such systems depends on the denominator polynomial (neglecting the fact that numerator polynomial can cause instability). Root maps of the polynomial must lie inside the unit circle. This can be checked numerically by a very fast and efficient method simply based on sampling the denominator polynomial using the fast Fourier transform. It was shown that this method is dramatically faster than one proposed in the literature based on the semi-definite programming expression. In spite of the fact that the proposed method provides necessary but not sufficient condition it can find applications in stability analysis of multidimensional systems, repetitive processes, or spatially distributed systems. The number of variables is not limited since for test of positiveness on the unit n-circle the algorithm for n-dimensional FFT is available. The result was achieved by the members of the evaluated DCT team only and was published as follows:

***P. Augusta, P. Augustová P.*** A fast numerical test of multivariate polynomial positiveness with applications, Kybernetika, vol.54, 2 (2018), pp. 289-303.

**Iterative Learning Control for a class of spatially interconnected systems**. This result shows design of iterative learning control (ILC) for a class of spatially distributed systems. As example, deformable mirror is considered. The mirror is described by fourth-order partial differential equation. For control design purposes, the partial differential equation is discretised in both spatial and temporal variables using an unconditionally stable finite difference discretization scheme motivated by the well-known Crank–Nicolson method. The derived model has the form of a recurrence equation. So, it is transformed into state-space equations to use it for the ILC design. The ILC consists in finding controller parameters iteratively. The feasibility of the ILC design was confirmed by numerical simulations. The above described methods were applied to two different cases of the deformable mirror shape. The result was partly achieved via cooperation of the evaluated team members with the Institute of Control and Computation Engineering, University of Zielona Góra, Poland and Department of Electronics and Computer Science, University of Southampton, see shares bellow. Where not stated otherwise, the share of DCT team is 100%. Its details were published in the following papers:

**P. Augusta, B. Cichy, K. Galkowski, E. Rogers**. An unconditionally stable approximation of a circular flexible plate described by a fourth order partial differential equation, Proceedings of the 21st International Conference on Methods and Models in Automation & Robotics 2016, pp. 1039-1044, Międzyzdroje, Poland. **DCT team members research share estimate is 25%.**

***B. Cichy, P. Augusta, K. Galkowski, E. Rogers***. Iterative Learning Control for a class of spatially interconnected systems, Trends in Advanced Intelligent Control, Optimization and Automation, pp. 734-743, Eds: Mitkowski W., Kacprzyk J., Oprzedkiewicz K., P. Skruch, 2017. **DCT team members research share estimate is 20%.**

***B. Cichy, P. Augusta, K. Galkowski, A. Rauh, H. Aschemann H., E. Rogers, B. Rehák.*** Modeling and Iterative Learning Control of a Circular Deformable Mirror, IFAC-PapersOnLine. Volume 50, Is. 1, 20th IFAC World Congress, pp. 3117-3122, Eds: Dochain D., Henrion D., Peaucelle D., World Congress of the IFAC 2017, Toulouse, FR. **DCT team members research share estimate is 30%.**

***P. Augusta, B. Cichy, K. Galkowski, E. Rogers.*** A finite difference scheme for iterative learning control-oriented modelling of physical systems described by PDEs in polar coordinates, Proceedings of the 23rd International Symposium on Mathematical Theory of Networks and Systems (MTNS 2018), Hong Kong, 2018, pp. 659-665. **DCT team members research share estimate is 30%.**

## Research activity and characterisation of the main scientific results

The team members have (co)-authored 5 monographs published by international publishers, more than 110 journal papers, and about 40 contributions in proceedings of international conferences. Some of them are listed below.

The research in the department is conducted in the following areas:

**Mathematical logic (C. Noguera, T. Lávička, B. Grimau).** We have focused on the mathematical study of non-classical logics as a tool to formalize reasoning. More precisely, we have contributed to the following lines of research:

(i) *Abstract algebraic logic*, the subfield of mathematical logic that studies logical systems by considering the ways they may be associated to algebraic counterparts. We have considered a very general family of propositional logics with a reasonable notion of implication and their interplay with a generalized disjunction satisfying the proof by cases property and we have obtained several useful characterizations of semilinearity (the essential property of fuzzy logics) in terms of disjunctions [9,10], and we have characterized completeness properties of logical systems with respect to general algebraic semantics, in particular, with respect to linearly densely ordered models [5,11]. Finally, we have proposed a novel extension of abstract algebraic logic to infinitary logics, i.e. systems with infinitary rules, in the master and PhD theses of Tomáš Lávička (supervised by Noguera) and in the papers [19,20].

(ii) *Predicate non-classical logics and their model theory.* Building on those propositional foundations, we have moved to the first-order formalism. We have given a uniform framework and extended Henkin's proof of completeness of classical first-order logic to the realm of algebraizable logics [8]. Based on this result, we have focused on the model theory of non-classical predicate logics: we have studied homomorphisms and elementary equivalence in non-classical first-order structures and proved very general forms of Löwenheim-Skolem theorems [15], we have given sufficient conditions for elementary equivalence in terms of back-and-forth systems [16], we have studied Fraïssé limits [3] and saturated models [2], and obtained syntactical characterizations of classes of models [1].

(iii) *Modal fuzzy logics.*
We have continued the study of fuzzy logics, a kind of many-valued logics with a remarkable application potential in representing gradedness and vagueness. In this period, we have focused on systems with higher expressivity given by modal operators. We have proposed and studied a new neighborhood semantics, generalizing usual Kripke frames, for many-valued modal logics [7,12,13]. Also, we have considered a two-layered syntax that turns out to be useful for the management of uncertainty, and we have described the translations between these systems and other logics of uncertainty proposed by Halpern et al [4]. Moreover, Noguera has co-edited a handbook collecting and systematizing the modern research on mathematical fuzzy logic [6].

(iv) *Formalization of reasoning in natural language.*
We have developed a framework for reasoning with graded predicates that approach combines Smith’s account of vagueness as a matter of degree and the algebraic

apparatus of mathematical fuzzy logic [14]. Also, Grimau has written a paper in which she provides evidence from natural languages for the legitimacy of a formalism called "Higher-Level Plural Logic", an interpreted formal language that extends Plural Logic by including higher-levels of plural terms and quantifiers and which has a number of interesting applications in various foundational projects, both in the philosophy of mathematics and in metaphysics [17]. Finally, Grimau has published a review of a book on the topic of logical analysis of reasoning in natural language [18].

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**Conditional independence structures and game theory (M. Studený, T. Kroupa, V. Kratochvíl).**

There were three areas of research during the observed period. The first topic was the role of supermodular functions in game theory and conditional independence structure description. Two papers were devoted to this topic. In 2016 paper [1] a criterion was proposed to recognize the extremity of a supermodular function. The new criterion gives a necessary and sufficient condition for the extremity and can be easily implemented on a computer. It is based on a different principle/idea than previous (partial) criteria, namely on the concept of combinatorial structure a core polytope (from game theory). The joint paper [2] with foreign colleagues (J. Cussens, D. Haws) revealed the role of extreme supermodular functions in the context of learning graphical models of conditional independence structure: these were shown to correspond to important score-equivalent facets of polytopes used for learning Bayesian networks. The second area was learning decomposable graphical models. In the joint paper [3] with J. Cussens an important class of clutter inequalities was introduced for the so-called chordal graph polytope to be used in learning these graphical models. These inequalities were shown be facet-defining for the polytope and a challenging conjecture was raised that they fully characterize the polytope. The third area was specifically the cooperative game theory. The paper [4] was devoted to the (polyhedral cone) of exact games which involves the cone of supermodular games. A criterion was proposed there to recognize the extremity of an exact game, which is based on analogous idea as the criterion for supermodular games from [1]. Both criteria were implemented on a computer. The paper [5] then brings a characterization of facets of the cone of totally balanced games and a conjecture about what are facets of the cone of exact games.

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**Information theory (F. Matúš).** The joint conference paper [1] of F. Matúš with I. Csiszár concerned the topic of minimization of integral functionals subject to infinitely many moment constraints. The result of cooperation of F. Matúš with L. Czirmaz was their 2016 paper [2] devoted to the cone of entropic functions. In the paper, the so-called four-atom conjecture (raised in secrete sharing community) on the minimization of Ingleton score was refuted. The 2018 paper [3] by F. Matúš was related to the theme of characterization of conditional independence structures induced by random vectors. Specifically, the paper offers a criterion to recognize conditional independence structures induced by binary random variables; it additionally describes the induced covariance sign structure.

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**Local structure in Bayesian networks (J. Vomlel).** Bayesian networks are models that encode certain class of Conditional Independence (CI) structures. In many practical applications, we can go one step further and restrict the class of models by considering a local structure within each conditional probability table. This is very useful since it simplifies not only the model building (since fewer parameters need to be estimated) but also computations with these models. We extended our previous results on efficient inference in Bayesian networks (BNs) exploiting CP tensor decomposition [1].

**Applications of Influence diagrams (J. Vomlel).** We applied Bayesian networks and their decision theoretic extensions - influence diagrams – to decision problems. We introduced influence diagrams to the problem of optimization of a vehicle speed profile [2]. Later, we generalized our approach so that it was possible to use it for solving other trajectory optimization problems [3].

**Computerized Adaptive Testing with Bayesian networks (J. Vomlel).** Another promising application area of Bayesian networks are computerized adaptive tests (CATs). We plan built on our previous work in this area. We designed Bayesian network models with special types of conditional probability tables (CPTs). These CPTs simplify the model-building phase and allow more efficient probabilistic

inference. We have designed and implemented learning of CPTs satisfying the monotonicity property [4].

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**Evidential compositional models (J. Vejnarová, V. Kratochvíl, R. Jiroušek).**
From 2015 through 2019 we have prolonged the research in the field of compositional models in direction to their applicability to problems of practice. Within the bilateral Czech-Taiwanese project we studied the possibility of using the methods of compositional-model learning to data-mining tasks. In the book [1], which was the main output of the project, and in paper [3], we focused on mining the knowledge that is encoded in compositional models, and which is legible to users from the field of application. This knowledge is either qualitative or quantitative. The former one can be characterized by the independence structure easily read from the model; the latter type of knowledge, the quantitative one, is based on the values of probabilities and/or conditional probabilities of states of the considered variables.
Another field of application arises from the fact that we succeeded to study causal models expressible in the form of compositional models, within both probability [4] and more general types of uncertainty theories [2].

Generally, not a small part of data-based model learning methods make use of tools and results from information theory. To make these principles available also in the alternative uncertainty theories, we contributed to the development of generalized information theories by proposing new ways of defining entropy in the Dempster-Shafer theory of evidence (D-S theory) [5] (note that the paper has 40 WoS citations within 15 months from publication). Criteria for finding optimal decisions are usually based on a maximum expected utility principle. Direct computation of the expected value in D-S theory is extremely time demanding. That is why we published a comprehensive comparison of various probabilistic transforms which can be easily computed and reasonably approximate the results yielded by the original approach.
The paper has appeared this year.

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**Variational analysis and its applications (L. Adam, M. Branda, M. Červinka, J. V. Outrata, M. Pištěk).**

(i) *Generalized differential calculus*

We substantially generalized some standard rules of the generalized differential calculus related to tangent and normal cones to sets having the so-called constraint structure [5] and to coderivatives of normal-cone mappings [1]. Further we have constructed a new calculus for directional limiting normal cones and subdifferentials which significantly differs from the classical Mordukhovich calculus. The new rules offer efficient tools for stability analysis of complicated equilibria [11]. Moreover, we tailored the basic concepts of variational analysis to the quasi-convex setting, see [17].

(ii) *Stability of parameterized equilibria*

For several classes of parameterized equilibria, we derived sufficient conditions ensuring various types of Lipschitzian stability with respect to small perturbations of the parameters. In particular, we studied (1) Problems with conic constraints [1], [2];
(2) Variational and quasi-variational inequalities [8], [13], [14];
(3) General equilibria modelled via implicit multifunctions [6], [14];
(4) Calculation of Normal Cones to a Union of Polyhedral Sets [16].

(iii) *Analysis and computation of complicated equilibrium problems*

We applied advanced tools of variational analysis to derive optimality conditions and to construct efficient numerical approaches for various (classes of) application-motivated equilibria. In particular, we considered mathematical programs with equilibrium constraints (MPECs) [7], [8]. Relaxation of qualification conditions is considered in [10]. Further, we analysed multiple optimization problems with equilibrium constraints (MOPECs) [4], relaxed reformulation of optimization problems with cardinality constraints [21], [22] (with application in portfolio optimization), relaxed reformulation of chance-constrained problems [18], [19] (with application in gas network design), and specific equilibria in deregulated electricity market in [15], [20].

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**Representation of intransitive preferences (M. Pištěk).**
In the domain of SSB representation of preferences we corrected and extended the theorem providing existence of maximal preferred element in an infinite set of alternatives [1]. We have also generalized the main theorem of the SSB representation, see [2].

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**Mathematical methods in mechanics of solids, calculus of variations, computational mechanics, and numerical analysis (S. Krömer, M. Kružík, J. Valdman, T. Roubíček, M. Kočvara, P. Gidoni).**
Numerous mathematical models in continuum mechanics and thermomechanics of solids described by systems of nonlinear partial differential equations or inequalities were formulated in a thermodynamically consistent way and mathematically analysed with the aim to prove existence of their solutions and devise efficient numerical strategies for their approximation implementable on computers. Examples are models in geophysics or in phase transformations in ferromagnetic materials and shape memory alloys [17,10]. Other instances include plasticity [1,22,23], damage, or adhesive contacts [20] formulated either in small or in large strains. The attention has also been paid to various activated, rate-independent/independent processes and to combination of different phenomena. Rate-independent processes and applications to various problems of continuum mechanics of solids are the main topic of the monograph [21]. On the other hand, the monograph [16] considers more general problems in large strain elasticity as well as in the linearized setting, including various rheologies and coupled thermodynamical processes. The focus on simple, as well as on nonsimle materials. A new class of nonsimple materials called gradient-polyconvexity was defined in [6].

One of the major open problems in nonlinear elasticity is a formulation of necessary and sufficient conditions for the existence of a solution subject to orientation-preservation and global injectivity constraints. This issue was finally resolved in [4] for the case of bi-Lipschitz maps in the plane. Various results on this topic are scrutinized in the survey paper [5].
Besides theoretical results on injective deformations, numerical analysis and computations are important for real-world applications. The paper [14] implements a fully practical numerical scheme ensuring the global injectivity.

A new research program has been opened on the justification of linearized models in solid mechanics. This is usually done by showing that the linearized version is a

suitable variational limit of the fully nonlinear problem. The paper [7] proves this in the case of viscoelastic solids. Two other papers on this topic are submitted.

A number of purely analytical problems has been studied, too. They deal wih fundamental questions of the calculus of variations. Here we have concentrated on problems of weak lower lower semicontinuity of integral functionals in nonreflexive spaces [2,3] or under general differential constraints [13].

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## Research activity and characterisation of the main scientific results

Unprecedented changes in the society we are witnessing past decades are revolutionizing decision making in an economy in ways that we only now are beginning to understand. Growing interconnectedness of economic agents, companies, machines, and computers form new dynamic networks that have changed the nature of economic market interactions. Trading has moved from physical markets to electronic marketplaces on giant computers or distributed networks and is increasingly executed by complex algorithms. Intervals between trades are now measured in fractions of seconds, and significant part of the agents trade at short horizons. Data are no longer stored on paper or on a single computer, but are mirrored, replicated, and perpetuated. Growing into dimensions that make traditional models inadequate, Big Data brings many challenges. Availability of huge and permanent information streams affect individuals, firms, countries, and the global trading system. Data has become a valuable economic resource, creating impulses for productivity improvement with uncertain economic impact on existing factors of production and the distribution of income among those factors. Large data sets generate both risks and rewards and are becoming central to economic decision making.
Key changes originate in information, communication and computing technologies. Yet its adoption, implementation, and societal impact are fundamentally economic processes: those that govern the allocation, production, and distribution of scarce resources. Textbook economic paradigms that assume the pre-eminence of scarce, physical and rivalrous goods are increasingly unsuitable for the analysis of goods and services. Economists are facing number of important challenges as decisions to invest involve lumpy decisions rather than one's subject to marginal calculus. Algorithms increase the productivity of existing production factors; they also threaten to crowd out or eliminate human, potentially irrational and slow decision makers. Traditional economics and finance both lost touch with many of these developments and needs to reorient its research.

Our research in the evaluated period reach across disciplinary lines to study several aspects shifting the traditional frameworks in the above context. The research areas the team contributes to form a natural structure as detailed below:

- Dynamic networks and financial decision making: Consumers, firms, and countries are creating ever intensifying linkages in a world economy. Monitoring these intimate network connections between many economic variables is central for risk measurement and management, central bankers, and policy makers. In series of papers, the team was interested in uncovering fundamental sources of network connections in which relationships vary over different horizons and time, and shock propagating over these horizons creates linkages. The team contributes to the literature with a novel measures of time-frequency dependent network connectedness [link] to measure networks arising due to heterogeneous responses to shocks at short, medium, or long run cycles. The methodology opens new routes for measurement of linkages and became one of the most cited papers in the journal. In addition, we document new stylized facts about cyclical properties of the transmission mechanism of the oil-based commodity markets using the network measures [link], and we study asymmetric networks arising due to different shocks in response to good and

bad news creating different uncertainty [link , link, link]. Further the team brings new methodology to forecast entire return distribution that is more informative in addition to commonly estimated mean and variance forecasts [link]. These contributions are important for financial decision making as well as understanding how real economy interacts with financial sector and most of these papers already attracted attention as top cited works of the journals.

- Dynamic quantile asset pricing models: Fundamental relation underlying all classical asset pricing is that the value of an asset is equal to its expected discounted payoff. Since investors must decide how much to save and how much to consume, and what portfolio of assets to hold, the pricing equation comes naturally from the first-order condition for this decision. The dynamic behavior of an economic agent is thus central to determining the value of assets. While the classical asset pricing frameworks is critical for asset valuation, they are all dominated by the expected utility models. Recently, many researchers find this idealization to be overly restrictive feeling that the expected utility model should be generalized. Collecting more and more data, and witnessing the shift in the behaviour of agents, new theoretical approaches need to be developed. One of the very recent routes in generalizing the concept is towards the change of classical preferences to quantile preferences. Growing empirical evidence that supports nontrivial deviations from expected utility-based model predictions calls for more general theoretical asset pricing. Specifically, it is tempting to develop a dynamic quantile asset pricing approaches that will better explain the behavior of and will provide a closer match with data we collect. In this line of research, the team contributed with several advancements. We provide simple non-parametric model for conditional quantile of asset returns [link], we also look at panel structure and factors in tails [link].

- Measurement of dependence between cyclical economic variables: The team introduces new general measures of dependence structures that remain invisible when only traditional analysis is employed [link] constituting significant contribution since it opens new routes for measurement of dependence in economic variables. In addition, we bring new methodology for quantifying common jumps in the currency markets and their impact of correlation structure [link]. Using frequency domain techniques the proposed framework uniquely localizes jumps and co-jumps very precisely and its usefulness is demonstrated on the empirical behavior of the most liquid currency markets. The work has been published in one of the leading journals publishing work about behavior of financial markets. The team further looks at modeling and forecasting exchange rate uncertainty using frequency domain techniques [link].

- Machine Learning/Statistical Learning in Economics: the team started the work that explores usefulness of machine learning techniques in economics and finance. We propose to model term structure of crude oil future prices with machine learning techniques [link] that constitutes significant contribution since it opens new routes for modeling term structure and it is a first study to explore this avenue that attracted lot of citations. In addition, we contribute with one of the first studies exploring machine learning techniques with high frequency data to improve modeling of volatility and uncertainty in stock markets [link].

- Understanding digital currencies the team also contributed to the fascinating phenomenon of digital finance, the team was among the first looking at potential drivers of bitcoin [link], while behavior of Bitcoin and Digital Finance attracted large literature recently, the paper attracted lot of attention with 465 citations.

## Research activity and characterisation of the main scientific results

The Image Processing team (ZOI team) was working in the 2015 – 2019 time period in five main research directions - the moment theory, image restoration, image forensics, cultural heritage applications, and medical imaging. The research was done in tight cooperation with both Master and PhD students. The research was supported from several grants, both theoretical as well as industry- oriented, funded by Czech Science Foundation (7), Charles University (4), Technology Agency CR (5, one of them is large National Competence Center program), GA UK (4), Ministry of Industry and Trade (2), Ministry of Interior (2), and Academy of Sciences CR (2). J. Flusser got the Academic Premium, which brought financial support, too. There are two international projects running in this time period, ALMARVI funded by ARTEMIS and PROVENANCE financed from H2020 scheme. Since 2015, ZOI has an active part in the activity of the Academy of Sciences of the CR called AV21, which brings as well financial support.

### Moment theory

The topic of moment theory and derived invariants with respect to both geometric and radiometric image degradations is well established in ZOI team and it has been still gaining significant attention. In 2016, the key book "*2D and 3D Image Analysis by Moments*" was published by three members of the team in *Wiley & Sons*. The major achievements in the area of moment invariants are the theory of projection operators and invariants to image blurring, design of new 3D geometric invariants, invariants for pattern detection in vector fields, contribution to the theory of orthogonal moments. In each category, the key papers were published in top journals of the field, including IEEE TPAMI (IF = 17.5), IEEE TIP and Pattern Recognition.

***Projection* Operators *and Moment Invariants to Image Blurring.*** We introduced invariants w.r.t. image blurring for broad families of blurring kernels, both in Fourier and moment domain. We introduced a notion of a primordial image as a canonical form of all blur-equivalent images. It is defined in spectral domain by means of projection operators. We proved that the moments of the primordial image are invariant to blur and we derived recursive formulae for their direct computation without actually constructing the primordial image. We published a series of papers which differ from one another by assumptions about the blurring functions. For N-fold symmetric kernels, the theory was presented in following journal (PAMI, IF = 17.5). The original theory improves the image recognition in real environment, particularly under an unknown blurring.

F*lusser J., Suk T., Boldyš J., Zitová B.: Projection Operators and Moment Invariants to Image Blurring, IEEE Transactions on Pattern Analysis and Machine Intelligence vol.37, 4 (2015), p. 786-802;(100%)*

For bluring function in the form of dihedral kernels

*Pedone M., Flusser J., Heikkila J.: Registration of Images with N-fold Dihedral Blur, IEEE Transactions on Image Processing vol.24, 3 (2015), p. 1036-1045 (40%)*

and for Gaussian blur kernels in (non-linear projection operators were used for the first time in this context to derive the invariants)

*Flusser J., Farokhi S., Höschl C., Suk T., Zitová B., Pedone M.: Recognition of Images Degraded by Gaussian Blur , IEEE Transactions on Image Processing vol.25, 2 (2016), p. 790-806 (90%)*

***Orthogonal moments in image analysis.*** OG polynomials and moments are known for their favourable numerical properties. This is why we have studied a possibility of their use in advanced image recognition. In the paper

*Yang B., Flusser J., Kautský J.: Rotation of 2D orthogonal polynomials, Pattern Recognition Letters vol.102,1 (2018), p. 44-49 (50%)*,

we proved a new necessary and sufficient condition under which a general 2D OG polynomial behaves in the same way as monomials under rotation. In this way we proved the prominent role of Hermite polynomials. This theoretical achievement was used in design of rotation invariants from Gaussian-Hermite moments

*Yang B., Suk T., Flusser J., Shi Z., Chen X. : Rotation invariants from Gaussian-Hermite moments of color images , Signal Processing vol.143, 1 (2018), p. 282-291, (50%)*

*Yang B., Flusser J., Suk T. : Design of high-order rotation invariants from Gaussian-Hermite moments*
*, Signal Processing vol.113, 1 (2015), p. 61-67 (70%)*

*Yang B., Kostková J., Flusser J., Suk T.: Scale invariants from Gaussian-Hermite moments, Signal Processing vol.132, 1 (2017), p. 77-84. (75%)*

*Yang B., Flusser J., Suk T.: 3D rotation invariants of Gaussian-Hermite moments , Pattern Recognition Letters vol.54, 1 (2015), p. 18-26. (100%)*

We proposed new OG polynomials with asymmetric weighting function

*Honarvar S. A. B., Flusser J.: New discrete orthogonal moments for signal analysis, Signal Processing vol.141, 1 (2017), p. 57-73 (100%)*

We paid a significant attention to computational issues and proposed new recursive schemes for 2D Zernike polynomials (published in the best-ranked journal in control engineering - IF = 6.4).

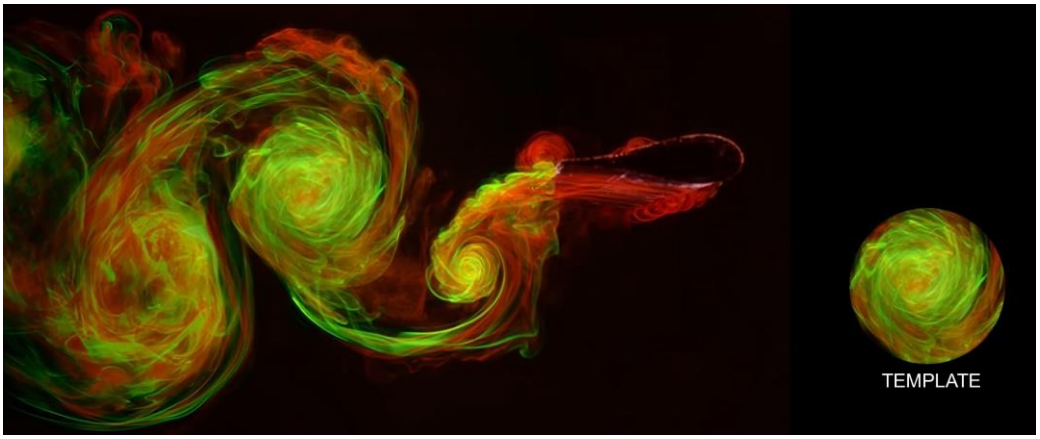
*Honarvar S. A. Barmak, Flusser J., Zhao Y., Erkoyuncu J. A.: Filter-generating system of Zernike polynomials , Automatica vol.108, 2019 DOI: 10.1016/j.automatica.2019.108498, (80%)*

***3D moments and their applications.*** In the last years, 3D imaging modalities have attracted an increasing interest of researchers and application developers. We focused our study on a design of a theory, which enables a consistent derivation of rotation invariants for image recognition. Published in one of the leading journals of the field (IF = 5.9).

*Suk T., Flusser J., Boldyš J.: 3D rotation invariants by complex moments , Pattern Recognition vol.48,11 (2015), p. 3516-3526.(100%)*

To speed up the moment calculation, we developed a decomposition algorithm which divides the object into a small number of rectangular blocks. The optimal solution of this problem has been known to be NP-complete but we achieved a close-to-optimal performance in a polynomial time

*Höschl C., Flusser J. : Close-too-optimal algorithm for rectangular decomposition of 3D shapes.,Kybernetika vol.55, 5 (2019), p. 755-781 (100%)*




***Algorithms for vector fields.*** Vector fields are a special kind of multidimensional data. In each pixel, the field is assigned to a vector or a matrix which shows the direction and the magnitude of the quantity, which has been measured. Vector fields appear in numerous scientific and engineering areas. We proposed two groups of pattern invariants, which can be used for detection of vortices, saddle points and other singularities.

*Yang B., Kostková J., Flusser J., Suk T., Bujack R.: Rotation invariants of vector fields from orthogonal moments, Pattern Recognition vol.74, 1 (2018), p. 110-121, (60%)* Published in one of the leading journals of the field (IF = 5.9).

*Kostková J., Suk T., Flusser J.: Affine Invariants of Vector Fields , IEEE Transactions on Pattern Analysis and Machine Intelligence DOI: 10.1109/TPAMI.2019.2951664. (100%)* Published in the best- ranked computer science journal ever (PAMI, IF = 17.5).

***Applications of moments.*** We successfully applied our theoretical results

- in face recognition
*Farokhi S., Sheikh U.U., Flusser J., Yang B.: Near infrared face recognition using Zernike momentsand Hermite kernels , Information Sciences vol.316, 1 (2015), p. 234-245,(25%)*

- astronomy
*Šimberová S., Suk T.: Dynamic process analysis by moments of extreme orders , Astronomy and Computing vol.14, p. 43-51 (2016) DOI: 10.1016/j.ascom.2016.01.003, (50%)*

- chemometric analysis of molecular shapes
*Honarvar S. A. Barmak, Flusser J.: Discrete Hermite moments and their application in chemometrics ,Chemometrics and Intelligent Laboratory Systems vol.177, 1 (2018), p. 83-88 (100%)*

- and in noise-robust image retrieval
*Höschl C., Flusser J.: Robust histogram-based image retrieval, Pattern Recognition Letters vol.69, 1 (2016), p. 72-81(100%)*

**Image Reconstruction**

***Deblurring*** is an active research topic in the ZOI team that contributed to 6 journal and 11 conference papers in the last five years, and was directly or indirectly supported by 4 grants. Deblurring was also the main topic of our collaboration with the AS team and international collaboration with Harvard University (Prof. You M. Lu). A benchmark dataset for evaluating deblurring methods was created and two simplified implementations of deblurring methods were implemented on smartphones.

We proposed a blind deconvolution method that handles seamlessly model discrepancies, such as overexposed areas in images, using a unified model for all priors that are adjusted on the fly from the iteratively estimated sharp image and blur. Decomposition methods were proposed that significantly speed-up space-variant deblurring.

*Kotera J., Šmídl V., Šroubek F.: Blind Deconvolution With Model Discrepancies , IEEE Transactions on Image Processing vol.26, 5 (2017), p. 2533-2544 (80%)*

*Šroubek F., Kamenický J., Lu Y. M.: Decomposition Space-Variant Blur in Image Deconvolution , IEEE Signal Processing Letters vol.23, 3 (2016), p. 346-350 (90%)*

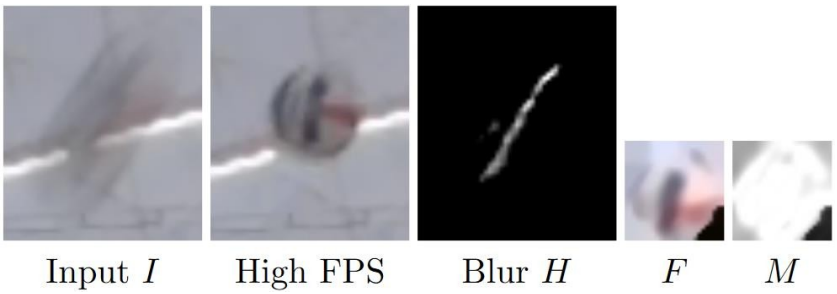



Figure Deblatting example. From left to right: the input image, corresponding high-speed camera frame, estimated blur H, estimated appearance F and shape M.

***Tracking of Fast Moving Objects*** is a new topic currently supported by a national grant investigated jointly with the Czech Technical University (Prof. J. Matas). So far, we managed to publish 5 conference papers including two A* conferences CVPR2017 and CVPR2020. We have introduced the problem of tracking fast moving objects. Such objects appear as semi-transparent streaks in videos and are common for example in sports videos, yet they are not present in standard banchmark datasets and most state-of-the-art trackers fail in this task. We proposed a tracking method based on anovel paradigm of tracking by deblatting - deblurring and matting – when an inverse problem of estimating the object trajectory, appearance and shape is solved per frame. More recently, learning- based approaches (CNN) are combined with deblatting to achieve more robust and also real-time performance.

*Rozumnyi D., Kotera J., Šroubek F., Novotný L., Matas J. : The World of Fast Moving Objects , 2017 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), p. 5203-5211, The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), (Honolulu, US, 20170721) (50%)*

***Image denoising.*** JPEG decompression can be understood as an image reconstruction problem and can be solved by iterative optimization algorithms. For many problems, if the sparse domain forms a tight frame, optimization by the alternating direction method of multipliers can be very efficient. Derivation of such solution is the main contribution of

*Šorel M., Bartoš M.: Fast Bayesian JPEG Decompression and Denoising With Tight Frame Priors , IEEE Transactions on Image Processing vol.26, 1 (2017), p. 490-501 (100%)*

Segmentation and denoising of signals often rely on the piecewise-polynomial model. Several signal denoising methods successfully combine the polynomial assumption with sparsity. In this work, we show that using orthogonal polynomials instead of other systems in the model brings numerical stability, better resolving of the breakpoints and removes the need for including additional parameters and their tuning.

*Novosadová M., Rajmic P., Šorel M.: Orthogonality is superiority in piecewise-polynomial signal segmentation and denoising , EURASIP Journal on Advances in Signal Processing vol.2019 (11%)*

For the convolutional sprase coding the speed-up was designed and published

*Šorel M., Šroubek F.: Fast convolutional sparse coding using matrix inversion lemma, Digital Signal Processing vol.55, 1 (2016), p. 44-51 (100%).*

**Image Forensic**

***Forgery detection.*** Copying an object and pasting in another location of the same image, is a common way to manipulate images. After saving to the JPEG format, pixels moved to a different position have different values, which generates many false detections in the majority of existing algorithms. To overcome this problem, we derive an efficient JPEG-based constraint that any pair of regions must satisfy to be considered a manipulation. The constraint can be integrated into most existing methods.

*Novozámský A., Šorel M.: Detection of Copy-move Image Modification Using JPEG CompressionModel, Forensic Science International vol.283, 1 (2018), p. 47-57 (100%)*

***Forensic analysis.*** The project PIZZARO resulted in the software tool, consisting of a set of methods for image and video forensic analysis. They were designed to help to assess image and video credibility and origin and to restore and increase image quality by diminishing unwanted artifacts (blur, noise). The motivation came from the best practices used in the criminal investigation. The methods were solved in tight cooperation of scientists from the Police of the Czech Republic and resulted in following project.

*Kamenický J., Bartoš M., Flusser J., Mahdian B., Kotera J., Novozámský A., Saic S., Šroubek F., Šorel M., Zita A., Zitová B., Šíma Z., Švarc P., Hořínek J.: PIZZARO: Forensic analysis and restoration of image and video data, Forensic Science International vol.264, 1 (2016), p. 153-166 (80%)*

Experience of ZOI team in the area of forgery detection and image doctoring analysis led to the succesful H2020 proposal PROVENANCE.

**Cultural heritage applications.**

We have continued our research in this interdisclipinary area. In 2015, the paper summing up our experience with microscopic cross-sections used for material analysis during art conservation was published. We present a performance evaluation of image

segmentation algorithms on microscopic image data. There is no universal method yet, thus, the issue of selecting suitable method for a given set of image data is of big interest. The benefit of segmentation combination approach was addressed,too. It was as well part of the doctoral thesis of M. Beneš.

*Beneš M., Zitová B.: Performance evaluation of image segmentation algorithms on microscopic imagedata, Journal of Microscopy vol.275, 1 (2015), p. 65-85 (100%)*

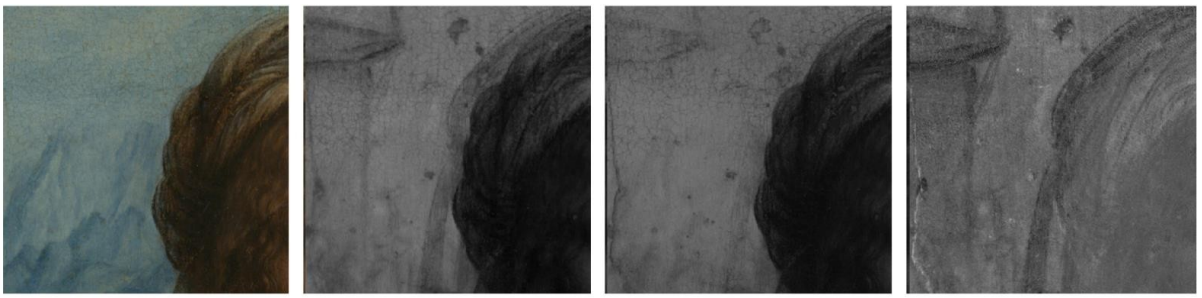


Figure. Pentimenti visualiation using deep neural networks. Our experiment with the16th century wooden desk painting attributed to Leonardo da Vinci. Left: RGB image, middle-left: a reflectogram centered at wavelength λ=1050nm, middle-right: output of ANN extrapolation, right: difference between the measured and the extrapolated outputs; the information gain.

Later on our focus shifted to the exploitation of deep learning methods for artwork analysis. We have developed an innovative digital processing methodology for accentuating information contained in the infrared reflectograms of old paintings (artworks). This novel image processing algorithm can be used for multimodal data comparison and is based on deep learning. Significant results of improvedvisualization are demonstrated on real paintings such as Madonna dei Fusi; Leonardo da Vinci in cooperation with INO – Italian National Institute of Optics, Florence, Italy.

*Blažek J., Striová J., Fontana R., Zitová B.: Improvement of the visibility of concealed features in artwork NIR reflectograms by information separation, Digital Signal Processing vol.60, 1 (2017), p. 140-151 (90%)*

Our cooperation with INO and University of Florence proved to be very fruitful and led to newmethodology for analysis of old paintings (Manet and Titian) using visual, near infrared and X-ray fluorescence data. Painting pigments were chemically and spatially characterized using spectral angle and correlation mapping methods. Novel artificial neural network algorithm was applied for improvement of visibility of pentimenti and underdrawings. The journal (IF 11.994) labeled the paperas HOT PAPER for its importance.

*Striová J., Ruberto C., Barucci M., Blažek J., Kunzelman D., Dal Fovo A., Pampaloni E., Fontana R. : Spectral imaging and archival data in analyzing the Madonna of the Rabbit painting by Manet and Titian , Angewandte Chemie - International Edition vol.57, 25 (2018), p. 7408-7412 (15%)*

An overview of our experiences we presented at International workshop on Image Processing for Art Investigation, organized by I. Daubechies (Duke Uni, USA), where the new cooperation was established with her team and the team of K. Janssens, Uni of Antwerp. We have now the bilateral cooperation with the later.

*Blažek J., Zitová B.: Information separation in art investigation - a survey, PROCEEDINGS of the sixth International workshop on Image Processing for Art Investigation (IP4AI 2018), p. 18-20, Image processing for art investigation, (Gent, Belgium) (100%)*

**Medical and biomedical imaging.**

In this area the ZOI team is exploiting its expert knowledge and uses it at various application areas. One of our biggest achievements is new and still ongoing collaboration with the Cambridge University, UK. Here we provide the visualization and analytic tools to the multidisciplinary research team analyzing the fundamental cellular process of endocytosis. Endocytosis was thought to be a spontaneous process, yet it was shown that it is regulated by several ways. The results were achieved by processing TIRF total internal reflection fluorescence microscopy images and can help develop new treatment strategies or anticancer agents. The IF of the journal is 9.1. A statistical analysis was carried out on hundreds of cells measured by a TIRF microscope. Our team was in charge of the automatic detection and tracking endocytosis in cells and estimating colocalization of different proteins regulating the cellular process.

*Wrobel A.G., Kadlecova Z., Kamenický J., Yang J.C., Herrmann T., Kelly B.T., McCoy A.J., Evans P.R., Martin S., Muller S., Šroubek F., Neuhaus D., Honing S., Owen D.J. : Temporal Ordering in Endocytic Clathrin-Coated Vesicle Formation via AP2 Phosphorylation , Developmental Cell vol.50, 4 (2019), p. 494-508 (20%)*

Another large area of interest, which newly started and already brought a grant funding, a study leave fror one of our colleagues and as well publications, is the analysis of tissue perfusion using magnetic resonance imaging (MRI). Quantification of perfusion and permeability of tissue on the capillary level is of great interest mainly in oncology, since it allows to discover functional changes before any morphological ones are observable. This field of interest is explored within a long-term cooperation with Institute of scientific instruments of the Czech Academy of Sciences, where technical equipment and related facilities are located. The role of ZOI is to utilize the state-of-the-art image and signal processing techniques to solve inverse problems present in this field. A traditional approach in quantification of perfusion using dynamic contrast-enhanced MRI is to fit a measured curve describing concentration of a contrast agent in time in each voxel to extract images of perfusion parameters of interest. Since the problem is non-convex and ill-conditioned, this approach of sequential fitting is imprecise. In our contribution, we extended the curve-fitting procedure to use

information from neighbouring voxels in terms of sparsity-inducing spatial regularization and presented an optimization algorithm to solve the problem.

*Bartoš M., Rajmic P., Šorel M., Mangová M., Keunen O., Jiřík R.: Spatially regularized estimation of the tissue homogeneity model parameters in DCE-MRI using proximal minimization, Magnetic Resonance in Medicine vol.82, 6 (2019), p. 2257-2272 60%*

We developed a blind deconvolution approach which estimates the necessary arterial input function from concentration-time curves in tissue voxels, which is followed by the estimation of perfusion parameters in all voxels.

*Jiřík R., Taxt T., Macíček O., Bartoš M., Kratochvíla J., Souček K., Dražanová E., Krátká L., Hampl A., Starčuk jr. Zenon: Blind deconvolution estimation of an arterial input function for small animal DCE- MRI , Magnetic Resonance Imaging vol.62, p. 46-56 (8%)*

We extended dynamic contrast-enhanced MRI model by dynamic susceptibility MRI model, which resulted in joint estimation of perfusion parameters from the simultaneously measured T1 and T2* signals to improve precision of the estimates.

*Macíček O.,Jiřík R., Mikulka J., Bartoš M., Šprláková A., Keřkovský M., Starčuk jr. Zenon, Bartušek K.,Taxt T.: Time-Efficient Perfusion Imaging Using DCE- and DSC-MRI, Measurement Science Review vol.18, 6 (2018), p. 262-271 (7%)*

Another example of an application of image processing methods in medical imaging is the capsule endoscopy video analysis, where the developed method helps to detect blood and thus eases the work of gastroenterologists. We proposed a new color space where the blood is optimally separated from other tissues and thus the possible regions of interest can be automatically detected.

*Novozámský A., Flusser J., Tachecí I., Sulík L., Bureš J., Krejcar O. : Automatic blood detection in capsule endoscopy video , Journal of Biomedical Optics vol.21, 12 (2016), p. 1-8 (80%)*

In the cooperation with breast oncologists we have introduced a system assisting during the breast ultrasonography (US) to cover the whole breast. The proposed system comprises a US device, electromagnetic 3D tracking technology and software combining visual and tracking data to estimate aprobe trajectory, time spent in different segments, and missed regions. The results of the case study indicate that missed regions are present in handheld whole-breast US, which renders the proposed system a valuable tool for monitoring coverage.

*Šroubek F., Bartoš M., Schier J., Bílková Z., Zitová B., Vydra J., Macová I., Daneš J., Lambert L.: A computer-assisted system for handheld whole-breast ultrasonography, International Journal of Computer Assisted Radiology and Surgery vol.14, 3 (2019), p. 509-516 (80%)*

The paper proposes novel method for computer assisted evaluation of diagnostically important vibration features, related to movements of vocal folds and their surroundings using videokymography, a high-speed medical imaging technique used in laryngology and offering important characteristics for diagnosis and treatment of voice disorders. Performance of the developed methods is compared to expert manual assessments. The method is already in use.

*Novozámský A., Sedlář J., Zita A., Šroubek F., Flusser J., Švec J. G., Vydrová J., Zitová B.: Image Analysis of Videokymographic Data, Proceedings of the 2015 IEEE International Conference on Image Processing, ICIP 2015, p. 78-82, IEEE International Conference on Image Processing 2015, (40%)*

The aim of the paper was to increase the quality of the quantification of isolated islets for clinical islet transplantation and to remove the necessity of a human operator. Two machine learning algorithms forislet quantification were developed, the trainable islet algorithm (TIA) and the non-trainable purity algorithm (NPA). These algorithms automatically segment pancreatic islets and exocrine tissue on microscopic images in order to count individual islets, and calculate islet volume and purity.

*Habart D., Švihlík J., Schier J., Cahová M., Girman P., Zacharovová K., Berková Z., Kříž J., Fabryová E., Kosinová L., Papáčková Z., Kybic J., Saudek F.: Automated Analysis of Microscopic Images of Isolated Pancreatic Islets , Cell Transplantation vol.25, 12 (2016), p. 2145-2156 (3%)*

Many of published papers were written together with international colleagues as well there are many publications realized together with people from other UTIA departments and other Academia institutes.

## Research activity and characterisation of the main scientific results

### 1 Visual appearance of surface material modelling

We have developed six new compound Markov random (CMRF) field-based models and one mixture-based model for the most advanced recent representation of visual properties of surface materials - the bidirectional texture function. These BTF-CMRF models belong to the recent state-of-the-art mathematical models for visual textures. One model combines a non-parametric control random field with local three-dimensional moving average random field models [29]. Two other models use parametric Bernoulli or Gaussian mixture control random field with local three-dimensional causal autoregressive models [40]. Another two BTF-CMRF models efficiently fuse a non-parametric random field model with several parametric Markovian random field models [30,31]. The BTF-CMRF model is generated by the Potts random field model build on top of the adjacency graph [32] of a measured mosaic and combined with the local wide-sense Markov model for local graph cells.

We developed an atypical three-dimensional, spatial Gaussian mixture BTF model in [39]. This model is exceptionally well suited for multispectral textile textures. Another model [63] we derived for color dynamic textures modeling, editing, or inpainting. For all developed BTF / dynamic texture models, we derived robust parameter estimation and efficient synthesis methods, which allow unlimited size realistic and high-quality material modeling and simultaneously an unbeatable compression ratio.

We combined these BTF models with shape modeling also for 3D cultural heritage objects virtual reconstruction [43, 44]. Surface material recognition dependence on the illumination and viewing conditions was studied in [58] using our measured BTF wood database and deep convolutional neural network (CNN) for supervised material recognition.

### 2 Visual appearance precise measuring

Reference materials measurement - Our state-of-the-art reference goniometric material appearance capturing approach applied to effect coatings was presented in [08]. We proposed a public data format for multi-angle image measurements in [10]. We provided captured materials from MAM database in this format at our website http://btf.utia.cas.cz/.
An approach to adaptive anisotropic BRDF measurement using a pre-computed basis of isotropic BRDFs from the UTIA BRDF database for captured BRDF reconstruction from sparse samples [70]. Additionally, we proposed an adaptive strategy for BRDF measurement using one-dimensional slices, forming a sparse four-dimensional structure. They can be measured by continuous movements of a light source and a sensor [67], [68]. Further, we suggested an adaptive approach of capturing extremely reflective materials and performed pure silver measurement in [07].

We presented a method of BRDF reconstruction from one-dimensional slices forming a sparse set of bidirectional samples. These samples were obtained by a portable measurement device [69]. Further, we presented a fast approach to approximate capturing of anisotropic BRDF using reflectors and a projector [22].

The BTF measurement's usefulness to the acquisition of polychromatic effect pigment was demonstrated in [55]. Next, we extended our adaptive measurement approaches to capture the texture of effect coatings. We tested four variants of half-difference parameterization and in psychophysical studies identified minimal sampling along dimensions of this parametrization [12].

Image-based analysis of effect coatings - We proved that image-based methods are well suited for characterization of effect coatings allowing instant discrimination between different coating systems and effect pigments [05],[13], and for a non-invasive automatic texture-based particle orientation analysis method [17]. Further, we applied a PCA-based minimal sampling approach to reconstruct dense measurement geometries based on a sparse set of measured samples, and optimized sparse directions for specific pigment types or coating systems [14]. The image-based method for cloudiness detection in effect coatings was presented in [13]. In [18], we analyzed perceptual differences between static and dynamic sparkle effect in effect coatings.

We used our benchmark BRDF database for BRDF modeling and analysis of visual anisotropy. We developed a novel method of detection of anisotropic material properties using a straightforward setup and introduction of the concept of anisotropic stencils predicting the location of anisotropic highlights [04]. This concept was later used for the fast fitting of anisotropic BRDF without the need for non-reliable iterative optimization. This model was first published at CGI 2015 conference [02] and its extended version in [01]. We studied visual effects of material anisotropy by psychophysical analysis of the human perception of BRDF anisotropy suggesting a new measure of visual anisotropy [03].

Analysis of interactions between material appearance and shape - We also continued to run user studies to identify a proper shape optimized for identification differences between BRDF. This shape can be used for the assessment of BRDF models' visual quality [48], [11]. Using an anisotropic BRDF model, we have shown that anisotropic appearance improves a car body's visual impression over the isotropic one [19], [09]. We extended this work to real materials. In [16], we studied the visual attractiveness of anisotropic coatings using magnetic pigment on a car-like shape for different viewing angles. In [06], we linked the perception of macroscopic directionality introduced through 3D printing with the perception of anisotropic reflectance effect it caused.

Understanding the perception of material appearance - Material structure visibility was studied through a set of psychophysical studies over BRDF and BTF data, and its computational predictor was introduced in [45]. Similarly, we studied material visual qualities on materials from the department BRDF database and evaluated the relationship between the visual, tactile, and subjective material qualities and material categories [20].

## 3 Supervised and unsupervised classification

We developed a new method of occlusion detection on eye images, [41] generalized the Daugman integrodifferential operator, and created the ground truth iris occlusions mask database for the iris research community. The method - Unsupervised detection of non-iris occlusions [41] on challenging eye images ranked first among 97

participants in the Noisy Iris Challenge Evaluation Contest. It was subsequently used as the best segmentation algorithm for generating ground truth masks for the MICHE II (as Mobile Iris Challenge Evaluation II) worldwide contest.

Another unsupervised method using Markovian illumination invariant features and multiscale mixture-based feature model was introduced in [42] and favorably compared with eight leading alternative published segmenters.

A supervised retrieval of wood veneers using a smartphone with significant improvement over alternative textural features was presented in [45]. A first successful supervised classifier of coniferous trees needles taxonomy based on the Jeffreys divergence was published in [34]. A breast density classifier published in [60] used very efficient learning and a small number of features to achieve comparable results with alternative methods. We developed an automatic system for recognizing endangered spur-thighed tortoise [65] using our proposed discriminative plastron features. Classification of the handwritten numerals using the probabilistic neural networks is analyzed in [25]. The computer network security models were published in [54, 58].

In the paper [26], we compare two approaches to probabilistic neural networks - the first one based on the mixtures of product components and the second one using the mixtures of dependence-tree distributions. By considering the concept of the dependent tree, we explicitly described the statistical relationships between pairs of variables at the level of individual components, and therefore, the resulting mixture's approximation power may increase substantially. We have found that the statistical contribution of the dependence-tree structures decreases in the course of EM iterations and tends to reduce to a simple product mixture model.

## 4 Feature selection

We used an advanced feature selection method to identify sparse bidirectional reflectance measurements for proper classification of main material categories like fabric, leather, and wood [15]. Multiple Instance Learning from samples represented by multiple feature vectors is developed in [50]. We presented three techniques for simultaneous visualization of samples, features, and multi-classes based on linear regression and matrix factorization [53]. The proposed visualization methods allow quick visual assessment of the feature's information content and their discriminability in addition to the assessment of multi-labels.

## 5 Illumination and geometrical invariants

Prevailing surface material recognition methods are based on textural features, but most of these features are very sensitive to scale variations, and the recognition accuracy significantly declines with scale incompatibility. We developed Markovian illumination invariants (derived from a 2D Markovian model) together with their scale sensitivity in [38] and compared them favorably to the most common alternatives, such as Local Binary Patterns, Gabor features, or histogram-based approaches. Markovian illumination invariants derived from the 3D Markov random field were introduced in [42]

and were tested on the BTF dataset from our Prague texture segmentation benchmark. Markovian illumination invariants derived from the 2D Markov random field were successfully applied [45] for supervised retrieval of wood veneers using a smartphone.

Spiral and rotational invariant Markovian models were introduced in [59, 62] and verified on all four public bark databases (AFF, Trunk12, BarkTex, BarNet). Our classifiers based on these model-derived features outperform the state-of-the-art alternative methods, including the convolutional neural networks ResNet, even on the by far largest public bark database BarkNet which contains 23 000 high-resolution images from 23 different tree species.

## 6 Markov random fields and mixture models

We derived six new compound Markov random field-based models [29, 30, 31, 32, 40] and a mixture-based model [39] for the most advanced recent representation of visual properties of surface materials - the bidirectional texture function. These models were applied to visual texture representation and modeling and for both supervised and unsupervised classification. The 3D mixture model was also applied to simultaneous noisy and scratched color texture restoration [33]. Another application of the Markovian model was for enhancing suspicious breast tissue abnormalities, such as microcalcifications and masses, to make signs of developing cancer better visually discernible, in digital X-ray mammograms [35, 37] for computer-aided diagnostic systems. We extracted over 200 local textural features from different frequency bands, which are then combined into a single multichannel image using the Karhunen–Loeve transform.

In the paper [24], we summarized our experience in applying finite mixtures with product components. We recall that the model does not imply the independence of variables, is not restrictive if the number of components is large enough, and increases the numerical stability of the EM algorithm. The model has some other essential advantages like efficient processing of incomplete data, information-based sequential decision-making, and structurally optimized mixture model.

## 7 Classifier and modelling quality benchmarking

We published [28] the results of the worldwide competition in unsupervised color image segmentation we organized, based on performance assessment using our on-line Prague texture segmentation benchmark and verification methodology together with one of our top unsupervised segmenters. This benchmark was extended with the set of randomly generated test mosaics using natural wooden BTF measurements from our extensive BTF database. We also developed the remote sensing data on-line benchmark [56] with a wide range of verification and ranking assessment methods and compared several leading published or commercial remote sensing classifiers. This benchmark uses ten spectral Advanced Land Imager satellite images and very-high-resolution GeoEye RGB images, with optional additive-noise-resistance checking and become a standard for remote sensing classifiers testing.

Visual texture fidelity evaluation is an essential but still unsolved problem. We proposed [51,52] two new texture fidelity criteria based on the fully multi-spectral generative underlying Markovian texture models, which correlates well with human

texture quality ranking verified on the texture fidelity benchmark. A unique dynamic texture similarity criterion based on the Fourier transformation and properties of dynamic texture spatio-temporal frequencies was proposed in [64] and validated using a sizable psychophysical test.

We proposed four new texture spectral similarity criteria [36, 46, 47] capable of assessing spectral modeling plausibility of color, bidirectional texture functions (BTF), and hyperspectral textures, which simultaneously consider the pixels of similar values and their mutual ratios. The performance quality of the criteria was examined in a long series of thousands specially designed monotonically degrading experiments, where proposed ones outperform fifteen tested alternatives. We developed a new contrast measure for measuring the local contrast of regions of interest in [61].

We analyzed to what extent the computer graphics representation conveyed the experience of viewing physical material appearance, and which material attributes and material types are judged consistently [10].

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## Research activity and characterisation of the main scientific results

### Recursive clustering algorithms

The main results during the time period from 2015 to 2019 were concerned with the extension of the Bayesian mixture estimation theory in the form of the development of recursive clustering algorithms and their software implementation in an open source and free environment for engineering computations. The main emphasis has been done on combinations of various types of the pointer model and mixture components to cover as much types of specific data as possible not limited by the normality assumption. The proposed clustering algorithms have the unified form allowing us the choice of prior probability density functions (pdfs) and their reproducible statistics updates depending on the component type, which facilitates a further extension of the methodology for other types of data.

The main contributions of the proposed algorithms are: (i) the recursive non-iterative data analysis in real time, (ii) covering specific non-Gaussian data types, (iii) dynamic switching the components, (iv) the unified structure of the algorithms, (v) the open source implementation. In addition to the most commonly used Gaussian mixtures [1]-[6], the following pdfs have been considered as the components: categorical [1,2], state-space [2], uniform [3] and exponential components [4] in the combination of the dynamic data-dependent pointer [1] or with the two-layer mixture model [5]. The initialization techniques have been considered as well [3]. The open source software packages have been prepared for a real-time non-iterative data-mining in Scilab (www.scilab.org), ready for use and adaptable to specific user tasks [5]. Real data sets from the transportation and medicine areas have been used for testing the proposed algorithms.

### Results reached in 2015-2019

[1] Suzdaleva E., Nagy I. An online estimation of driving style using data-dependent pointer model, Transportation Research. Part C: Emerging Technologies vol.86, 1 (2018), p. 23-36.

[2] Nagy I., Suzdaleva E. Algorithms and Programs of Dynamic Mixture Estimation. Unified Approach to Different Types of Components, Springer, (Cham 2017) SpringerBriefs in Statistics.

[3] Suzdaleva E., Nagy I., Pecherková P., Likhonina R. Initialization of recursive mixture-based clustering with uniform components, in Proceedings of the 14th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2017), p. 449-458, Madrid, ES.

[4] Nagy I., Suzdaleva E., Pecherková P.: Comparison of various definitions of proximity in mixture estimation, in Proceedings of the 13th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2016), p. 527-534, Lisbon, PT.

[5] Suzdaleva E., Nagy I. Two-layer pointer model of driving style depending on the driving environment, Transportation Research. Part B: Methodological vol.128, 1 (2019), p. 254-270.

[6] Nagy I., Suzdaleva E., Pecherková P. Clustering and Classification Using Recursive Mixture Estimation. Software package, ( 2018).

## Recursive identification, linear regression and systolic arrays

We develop and implement recursive system identification algorithms based on the Bayesian theory. In case of linear regression model, the Bayesian framework leads to algorithmic extensions of the family of recursive least square algorithms (RLS). The implemented extensions are base for systematic formulations of parameter tracking approaches and for the normalized fixed-point implementations of adaptive algorithms.

We implement recursive updates of the information matrix with numerical robustness in a square root, factorized forms by sequences of orthogonal rotations. These factorized recursive least square algorithms, known as QR RLS are mapped into pipelined triangular structures known as systolic arrays. Multiple subsequent time steps of the recursive identification are executable in parallel on the same array of computing cells. This brings massive data parallelism at the algorithmic level. This is utilized in our effective implementations of these algorithms in hardware.

In signal processing applications, the system-model is often defined as a finite response regression model (FIR). This brings additional properties to the recursively updated factorized matrices. The corresponding pipelined systolic arrays are simplified to the QR RLS Lattice structures, with the same numerical properties as the QR RLS but the numerical complexity of QR RLS Lattice grows only linearly with the system order. We implement versions of these recursive signal processing algorithms which eliminate the need of scalar square root operation known as QRD.

The recursive Bayesian framework guides us in new definitions of recursive QRD signal processing algorithms, which are capable to recursively estimate and track the probabilities of a fix set of hypotheses about structure of the regression model. In general, it requires to computation of one QRD model for each hypothesis. However, the pipelined QRD systolic arrays and the QRD Lattice arrays contain, under conditions defined in the recursive Bayesian framework, all information needed for the evaluation of all lower-order, nested, regression models in the maximal size systolic array. If the application constrains allows utilizing of this nesting property, the estimation of probabilities of the optimal system order can be realized, without the penalty of recursive identification of all QRD or QRD Lattice models in parallel.

The Bayesian framework helps us to extend the concept of nested lower order models and to derive theoretically the corresponding probability distributions of all transformed data propagated in the pipelined systolic QRD and QRD Lattice arrays.

Special deterministic transformations of QRD algorithms maintain the nesting properties and the pipelined systolic structure of the array needed for effective hardware implementation. Additionally, all data propagated through the pipelined systolic array and all the transformed coefficients have probability distributions with the

guarantied data ranges <-1, 1>. This opens the possibility to implement all computation of the algorithm only in fixed-point arithmetic.

The use of fixed point implementation leads to low power consumption and low latency of operations. Overflows at the boundary range <-1, 1> are purely the consequence of the limited precision of data representation and can be solved by use of saturation hardware in all performed fixed-point arithmetic operations. The Bayesian framework gives us the guidance and theoretical base for these regularizations implementable effectively in hardware.

## Results reached in 2015-2019

[7] Adaptive RLS Algorithms Reference Implementations
http://sp.utia.cz/index.php?ids=results&id=dsp_1_6

In frame of EU project SILENSE, we have developed, documented and released evaluation package [7] with reference implementations present set of adaptive recursive least squares system identification algorithms based on the Bayesian extensions of real-time adaptive system identification as well as extending the existing recursive least square adaptive algorithms for estimation of time varying parameters in the applications of acoustic signal processing. The released reference package of adaptive algorithms serves as "golden" reference models for the embedded implementations on dedicated processors like the 32 bit Arm Cortex A9 and the 64 bit Arm Cortex A53 and the FPGA programmable logic accelerators in Xilinx Zynq and Zynq Ultrascale+ devices. Algorithms are numerically robust. Algorithms are implemented in double precision floating point (64bit), single precision floating point (32bit) and in logarithmic arithmetic with precision 32bit and 19bit. All algorithms except for lattice filter are implemented both with exponential forgetting and directional forgetting. The released reference package also serves as "golden" reference for the adaptive recursive least squares system identification algorithms taking advantage of dynamic normalization of the core of the algorithm into the guarantied range <-1, 1> for all variables. These algorithmic cores are suitable for the fixed-point implementation with 14 bit fixed point arithmetic. It serves as reference for the ultralow power implementation of recursive RLS on parallel HW accelerators with custom fixed point arithmetic.

[8] Noise Cancellation Using QRD RLS Algorithms
http://sp.utia.cz/index.php?ids=results&id=noise-cancellation

In frame of EU project SILENSE, we have released evaluation package [8]. It serves for pre-processing of data for the final gesture-recognition application. In applications for gesture recognition the signals can reflect and be detected not only from a desired source (a hand), but from the environment as well, which creates undesired noise and hardens the process of precise gesture identification. Released package shows advantages and disadvantages of existing alternative approaches which are useable for the noise cancellation.

[9] INDUSTRIAL 40 NM DEMONSTRATOR NUCLEO STM32H755ZI-Q
http://sp.utia.cz/index.php?ids=results&id=H755ZI-Q

In frame of EU ECSEL project WAKeMeUP (project is coordinated by STMicroelectronics FR), we have documented and released evaluation package of industrial demonstrator [9] with implementation of adaptive signal processing QRD algorithms on family of 40nm STM32H7 microcontrollers on STMicroelectronics NUCLEO evaluation boards. Adaptive algorithms are specified in Scilab interpret application running on the ArduZynq shield on Xilinx Zynq device. The C code is generated from by our Scilab scripts as source code for implementations of the system identification algorithm on the STM32H7 microcontroller. Algorithm is compiled in our SW project compatible with the STM32Cube development framework. This UTIA result served as the WAKeMeUP project Milestone. The Milestone is documenting the maturity of family of the general purpose STM32H7 chips with the 2MB embedded flash developed in the WAKeMeUP project. Project WAKeMeUP is considered by the ECSEL JU as one of the key pilot-line EU projects securing the capability of manufacturing of advanced microcontrollers in Europe (In production plant of STMicroelectronics in Croles, near Grenoble in France).

## Design and implementation of signal processing in FPGA

We implement signal processing algorithms in form of effective hardware computing blocks known as intellectual property cores (IP cores) for field programmable gate arrays (FPGA). FPGAs allow us to develop, design and implement application specific integrated circuits (ASICs) for signal processing. FPGAs contain the programmable logic resources like the optimized, configurable, fixed-point multiply and accumulate (MAC) IP blocks and the optimized, random-access memory (RAM) IP blocks.

### Results reached in 2015-2019

[10] SILENSE TE0706+TE0720 Ultrasound Capture Platform with Example Application
http://sp.utia.cz/index.php?ids=results&id=capture_platform

[11] UTIA evBoard v1.0 Beamforming Demo
http://sp.utia.cz/index.php?ids=results&id=evBoard_v10

[12] UTIA Evaluation Board v1.7 - v1.8 Beamforming Demo
http://sp.utia.cz/index.php?ids=results&id=evBoard_v17_v18

In frame of EU project SILENSE (project is coordinated by NXP Belgium), we have developed, documented and released hardware evaluation board packages [10], [11], [12] performing 2D beamforming and serving for the ultras-und-based hand-gesture tracking. Our team developed and implemented all hardware interfaces in FPGA and implemented the HW accelerated DSP chain for the 2D beamforming application. PC SW was developed for visualization of measured data. The realized hardware platform consists of three basic HW components: TE0720 FPGA industrial system on module with Xilinx Zynq device, the TE0706 carrier board and the UTIA board with two orthogonal microphone arrays. System developed by UTIA served as one of the main demonstrators of SILENSE project in the EU review of the project.

## Design and implementation of reprogrammable HW accelerators

The mapping, design and implementation of signal processing algorithms in form of systolic arrays on the FPGA hardware requires typically reuse of multiple parallel data paths with the pipelined arithmetic units (floating point arithmetic units or logarithmic arithmetic units with different word-lengths). This mapping can be realized in a fixed hardware by several identical synchronous finite state machines sharing the arithmetic data paths in different, time-shifted, synchronous execution-slots.

Our team developed and implemented the EdkDSP platform. The EdkDSP is replacing these parallel, finite-state machines, by single-instruction, multiple-data (SIMD) architecture, with run-time reprogrammable controller. The EdkDSP platform enables to design relatively complex FPGA IP cores which can be connected to the standard processor and reprogrammed in the runtime of the application by change of firmware of the EdkDSP internal controller. The EdkDSP IP core is fixed hardware instance. However, it implements a family of signal processing algorithms.

In the reported period 2015-19, our implementations of the EdkDSP HW accelerator have been extended for use in system-on-chip devices with different processors and OS. At present, we support dual-core Arm A9 or quad-core A53 processor running Linux and the MicroBlaze soft-core processor with cache-based access to the DDR memory. Our implementations of EdkDSP HW accelerator use the asymmetric multiprocessing with HW supported synchronization of processors.

Modelling and design-flow adopted our team for the standard UTIA EdkDSP platform led to dependency on certain version of Xilinx design-tool chains. Only the Artix, Viretx and the 28nm Zynq 7000 families pf Xilinx devices have been supported.

To overcome this limitation, our team re-developed the EdkDSP platform into a new 8xSIMD run-time reprogrammable HW accelerator. It can target the 16nm Xilinx Zynq Ultrascale+ devices, now. We have also developed the streaming-data path with automatically generated HW data-movers (DMA or Scatter-Gather DMA). We have also developed and support the corresponding SW interface API for the Linux kernel drivers. The drivers come from company Xilinx for the 32 bit Arm A9 and the 64bit Arm A53 processors. Drivers are part of Xilinx PetaLinux kernel and Debian OS, now.

## Results reached in 2015-2019

[13] Isakovic H., Grosu R., Ratasich D., Kadlec Jiří, Pohl Zdeněk, Kerrison S. : A Survey of Hardware Technologies for Mixed-Critical Integration Explored in the Project EMC2 , Computer Safety, Reliability, and Security : SAFECOMP 2017 Workshops, ASSURE, DECSoS, SASSUR, TELERISE, and TIPS, p. 127-140 , Eds: Tonetta Stefano, Schoitsch Erwin, Bitsch Friedemann, SAFECOMP 2017 International Conference on Computer Safety, Reliability, and Security, (Trento, IT, 20170912) [2017]
http://library.utia.cas.cz/separaty/2017/ZS/kadlec-0479509.pdf

Conference paper [13] describes development made in the ECSEL EMC2 project. Project served as a large cross-disciplinary incubator for novel technologies and state-of the-art industrial applications. Paper describes the actual challenges for hardware

architectures and related technologies. UTIA team provided reference architecture based on Xilinx 28nm Zynq HW for several project partners. Developed systems are all using our run-time reconfigurable 8xSIMD EdkDSP accelerators.

[14] Full HD Video Processing in HW with three EdkDSP 8xSIMD Accelerators for TE0715-30-1 SoM on EMC2-DP-V2 Carrier
http://sp.utia.cz/index.php?ids=results&id=s30i1hm4

[15] Asymmetric Multiprocessing with MicroBlaze, EdkDSP Accelerator and Toshiba Sensor Video Processing for low cost Zynq on TE0720-03-1CF SoM on TE0701-05 Carrier
http://sp.utia.cz/index.php?ids=results&id=t20c1tm1

[16] Asymmetric Multiprocessing with MicroBlaze, EdkDSP Accelerator and Toshiba Sensor Video for Automotive grade Zynq on TE0720-03-1QF SoM on TE0701-05 Carrier
http://sp.utia.cz/index.php?ids=results&id=t20q1tm1

[17] Evaluation of Asymmetric Multiprocessing for Zynq System-on-Modules TE0720-02-2IF, TE0720-02-1CF, TE0720-02-1QF with Carrier Board TE0701-05
http://sp.utia.cz/results/emc2_amp_on_zynq_trenz_2015_2/Utia_EdkDSP_Vivado_2015_2_EMC2_te0720_te0701.pdf

In frame of EU project EMC2 (project is coordinated by Infineon Germany), we have developed, documented and released our EdkDSP accelerators [14], [15], [16], [17] for industrial-grade Zynq modules running the Xilinx PetaLinux operating system.

[18] Two serial connected evaluation versions of FP03x8 accelerators for TE0820-03-4EV-1E module on TE0701-06 carrier board
http://sp.utia.cz/index.php?ids=results&id=te0820_fp03x8x2s

[19]FP01x8 Accelerator on TE0726-03M
http://sp.utia.cz/index.php?ids=results&id=te0726_fp01x8

In frame of EU project FitOptiVis (project is coordinated by Philips, NL), we have developed, documented and released the new generation of our 8xSIMD HW accelerators for the Zynq Ultrascale+ [18] industrial-grade modules and for the small, entry-level system running on the low-cost ZynqBerry board [19].

[20] Evaluation version of 8xSIMD FP01x8 accelerator for ArduZynq shield
http://sp.utia.cz/index.php?ids=results&id=te0723_fp01x8

In frame of EU project WAKeMeUP (project is coordinated by STMicroelectronics FR), we have developed documented and released the new generation of UTIA 8xSIMD HW accelerators [20] for Zynq device on the small ArduZynq shield produced by company Trenz Electronics. The shield is compatible with the NUCLEO carrier board produced by company the company STMicroelectronics.

### Design and implementation of SW defined HW accelerators

In 2015 Xilinx introduced “Software Defined System on Chip”” design framework (SDSoC). The SDSoC framework is based in Xilinx “High Level Synthesis” compiler (HLS). SW C/C++ functions can be compiled into HW accelerators by the HLS compiler and the SDSoC framework generates HW data movers and the corresponding transformation of the SW project C/C++ code. The stand-alone SW and the Xilinx Petalinux SDSoC development platforms are supported by Xilinx, but only for the evaluation boards offered by Xilinx.

Our team developed support packages for the industrial modules and boards provided by other vendors. We specially support the industrial-grade modules developed and manufactured by the German company Trenz Electronic. These industrial modules are suitable for development of demonstrators for the ECSEL JU projects. Our team offers this support in form of documented design-time-resource packages. This solution is widely used by our partners in ECSEL projects.

## Results reached in 2015-2019

[21] SDSoC 2015.4 Standalone BSP with Full HD HDMI In-Out with SW and HW Demos for Zynq System-on-Module TE0715-03-30 and Sundance EMC2-DP-V2 Platform
http://sp.utia.cz/index.php?ids=results&id=s30i1h2

[22] Full HD HDMI In-Out HW-Accelerated Demos for Zynq System-on-Module TE0715-03-30-1I and Sundance EMC2-DP-V2 Platform
http://sp.utia.cz/index.php?ids=results&id=s30i1h1

[23] EMC2-DP HDMI in HDMI out Platform
http://sp.utia.cz/index.php?ids=results&id=emc2-dp-platform

In the frame of the EMC2 project our team developed, documented and released SW/HW packages [21], [22], [23] needed to enable the Xililinx SDSoC design framework for the EMC2-DP-V2 platform. This HW platform is manufactured by the company Sundance UK. It has been successfully demonstrated as the main result of the HW-architecture related work-package of the EMC2 project in the final review.

[24] Design Time and Run Time Resources for Zynq Ultrascale+ TE0820-03-4EV-1E with SDSoC 2018.2 Support
http://sp.utia.cz/index.php?ids=results&id=FitOptiVis-te0820-SDSoC-2018_2

[25] Design Time and Run Time Resources for Zynq Ultrascale+ TE0808-04-15EG-1EE with SDSoC 2018.2 Support
http://sp.utia.cz/index.php?ids=results&id=FitOptiVis-te0808-SDSoC-2018_2

[26] Design Time and Run Time Resources for the ZynqBerry Board TE0726-03M with SDSoC 2018.2 Support
http://sp.utia.cz/index.php?ids=results&id=FitOptiVis-te0726-SDSoC-2018_2

In frame of EU project FitOptiVis (project is coordinated by Philips, NL), we have developed, documented and released the design time support [24], [25], [26] for the

Xilinx SDSoC for medium-size and large Zynq Ultrascale+ systems with Full HD HDMI Video I/O and also for the small ZynqBerry board.

Our team leads the design-time-resources work package of the FitOptiVis project and our released packages are used as design-time resource by FitOptiVis partners.

[27] Hoozemans, J. - Van Straten, J. - Viitanen, T. - Tervo, A. - Kadlec, Jiří - Al-Ars, Z. ALMARVI Execution Platform: Heterogeneous Video Processing SoC Platform on FPGA. Journal of Signal Processing Systems for Signal Image and Video Technology. 91, no. 1 (2019), s. 61-73. ISSN 1939-8018
http://library.utia.cas.cz/separaty/2019/ZS/kadlec-0499963.pdf

Journal paper [27] describes a heterogeneous computational platform comprising of multiple communicating processors that allow easy programmability. ECSEL project ALMARVI addressed such processing elements. UTIA team designed platform on Xilinx Zynq industrial module with embedded Arm processor and programmable logic area, together with video sensor interface serving for HW accelerated video processing. System was used for application developments by ALMARVI partners.

[28] Zaid, A. - Basten, T. - Beer, A. - Geilen, M. - Goswami, D. - Jääskeläinen, P. - Kadlec, Jiří - Alejandro, M. - Palumbo, F. - Peeren, G. - Pomante, L. - Linden, F. - Saarinen, J. - Säntti, T. - Sau, C. - Zedda, M.
The FitOptiVis ECSEL project: highly efficient distributed embedded image/video processing in cyber-physical systems.
Proceedings of the 16th ACM International Conference on Computing Frontiers. New York: ACM Digital Library, 2019 - (Palumbo, F.; Becchi, M.; Schulz, M.; Sato, K.), s. 333-338. ISBN 978-1-4503-6685-4.
http://library.utia.cas.cz/separaty/2019/ZS/kadlec-0505799.pdf

Conference paper [28] describes research in ECSEL FitOptiVis project. Project aims to provide end-to-end multi-objective optimization for video pipelines, with emphasis on energy and performance. UTIA team provided reference architecture based on Xilinx 16nm Zynq Ultrascale+ devices for all project partners. The architecture supports Xilinx SDSoC system level compiler serving for compilation of C/C++ algorithms into HW accelerators with automatically generated HW data movers and data paths.

[29] Arrowhead Compatible Zynq Ultrascale+ Systems with Xilinx SDSoC 2018.2 Support
http://sp.utia.cz/index.php?ids=results&id=Zynq_Ultrascale

[30] Arrowhead client on Zynq 7000 device with support for the Xilinx SDSoC 2018.2 HW accelerators
http://sp.utia.cz/index.php?ids=results&id=ZynqBerry

In frame of EU project Productive 4.0 (project is coordinated by Infineon, D), we have developed, documented and released support for nine Arrowhead framework compatible Zynq Ultrascale+ systems [29] with support for the Xilinx SDSoC 2018.2 system level compiler. The Arrowhead framework is set of services developed by other partners in the project and serves for management of large systems-of-systems based on secure authentication and communication over internet.

Our team has developed and supports the C/C++ SW clients for Zynq devices, compatible to the Arrowhead framework. Clients can operate in secure mode.

Our Zynq Ultrascale+ systems accelerate computing by HW acceleration of algorithms in the programmable logic. The total system energy consumption associated with single iteration of an HW accelerated algorithm is significantly reduced in comparison to the pure SW implementation. The nine supported systems [29] use an identical Debian configuration. Systems are compatible with the Arrowhead framework. We also support the small, low-cost Zynq system [30].

[31] eMMC AXIS Controller Interfacing MTFC32GJWDQ-4M 32 GB Memory on Xilinx KC705 Board
http://sp.utia.cz/index.php?ids=results&id=emmc-axis-v1-doc-src-kc705

[32] eMMC AXIS Controller Interfacing MTFC32GJWDQ-4M 32 GB Memory on Xilinx ZC702 Board
http://sp.utia.cz/index.php?ids=results&id=emmc-axis-v1-doc-src-zc702

In frame of EU ENIAC project IDEAS (project is coordinated by IMA s.r.o with main industrial partners Micron, Italy and STMicroelectronics, Italy), our team prepared set of HW IP cores serving for interfacing of eMMC 32 GB non-volatile memory MTFC32GJWDQ-4M. This embedded nonvolatile memory is interfaced by an eMMC AXIS controller on Xilinx Kintex KC705 board [31] and Xilinx ZC702 FPGA board [32]. The controller is designed to support the MMC 4.51 standard. The application demonstrates the autonomous writing and reading of video data to the eMMC memory. The eMMC memory chips are located on the FMC expansion board. The board was designed and realized by our team in UTIA.

## R&D projects with Technical Development of Skoda Auto

Coordination of R&D projects with Technical Development of Skoda Auto is based on long-term contracts between UTIA and Skoda Auto. It has resulted in concrete development work performed in Department of signal processing, Department of adaptive systems and the Computer Centre with participation many external partners like National Institute of Mental Health. In mentioned projects was main attention devoted to analysis of HMI interaction based on vehicle control systems.

One of the main areas for enhancement cooperation in integration and testing vehicle systems are fully electric vehicles. This research is leading to a step change in the control software architecture with particular focus on comprehensive energy management. The main objective has been the energy savings and extended driving range of the fully electric vehicle, with benefits of improved vehicle safety and comfort. This research has been performed in the EU iCOMPOSE project, where ÚTIA has been acting as the sub-contractor to the project partner Skoda Auto or in the EU E-VECTOORC, where ÚTIA had acted also as sub-contractor for development of individual control of the electric motor torques.

Educational side of bilateral cooperation with Technical Development Skoda Auto is emphasized by integration students of different levels of study incl. doctoral study to the National and European R&D grant projects e.g. H2020 Marie Curie or KIC EU.

## Results reached in 2015-2019

[33] Herda Z., Nedoma P., Plíhal Jiří : The Design of Vehicle Speed Profile for Semi-autonomous Driving , Comprehensive Energy Management - Eco Routing & Velocity Profiles, p. 39-59 , Eds: Watzenig D., Brandstätter B. [2017] Download DOI: 10.1007/978-3-319-53165-6_3

Chapter in the book [33] provides findings of recent European project in FP-7 named “Intelligent Control of Multiple-Motor and Multiple-Storage Full Electric Vehicles” that were finally marked with sign excellent. The subject areas are pointed to prediction optimized speed profile in a dynamic environment and model-based eco-routing strategy for electric vehicles in large urban networks incl. SKODA Rapid ground tests modelled by the Bayes approach.

## Research activity and characterisation of the main scientific results

The team has carried out a fundamental research in the field of probability theory, mathematical statistics and statistical signal processing.
A. In probability theory, our research concentrates mainly (but not exclusively) on infinite dimensional systems (stochastic partial differential equations and interacting particle systems).
a) Research in the field of stochastic analysis:
a1) Existence of invariant measures for several types of stochastic partial differential equations, to which standard methods were not applicable, was established by studying Markov processes having the Feller property with respect to the bounded weak topology of the state space. In this way, existence of an invariant measure for a stochastic extensible beam equation was proved for the first time and results on existence of invariant measures for stochastic wave equations with polynomial nonlinearities (in both drift and diffusion) were considerably extended. The developed method turned out to be applicable also to (two-dimensional) stochastic Navier-Stokes equations in unbounded domains. See Z. Brzeźniak, M. Ondreját, J. Seidler, Journal of Differential Equations 260, 5 (2016), pp. 4157-4179, and Z. Brzeźniak, E. Motyl, M. Ondreját, Annals of Probability 45, 5 (2017), pp. 3145-3201.
a2) It seems that for stochastic wave equations with polynomial nonlinearities or for stochastic geometric wave equations, only few results on invariant measures are available apart from existence results. To show that the situation may be quite complex, a nontrivial toy model was analyzed in detail, which leads to a degenerate stochastic equation in the tangent bundle of a two-dimensional sphere. Non-uniqueness of invariant measures was proved and results about their domains of attraction were obtained. See L. Baňas, Z. Brzeźniak, M. Neklyudov, M. Ondreját, A. Prohl, Czechoslovak Mathematical Journal 65, 3 (2015), pp. 617-657.
a3) While existence of martingale (i.e., weak in the probabilistic sense) solutions to nonlinear stochastic Schrödinger equations driven by Lévy noise was known, uniqueness remained an open problem. Using Kurtz's abstract version of Yamada-Watanabe theory, we proved pathwise uniqueness, uniqueness in law and existence of strong solutions, see A. de Bouard, E. Hausenblas, M. Ondreját, Nodea-Nonlinear Differential Equations and Applications 26 (2019), paper. no 22, pp. 31.
a4) Space-time regularity (Hölder continuity) of solutions to linear stochastic partial differential equations driven by a cylindrical Volterra process was established, the driving process being neither Gaussian nor a semimartingale in general. (E.g., the case of fractional Brownian motion, Liouville fractional Brownian motion or a Rosenblatt process is covered.) See P. Čoupek, B. Maslowski, M. Ondreját, Stochastics and Dynamics 18, 6 (2018), paper no 1850048, 22 pp..
a5) The continuous-time Robbins-Monro stochastic approximation procedure was extended to infinite dimensional Hilbert spaces. Sufficient conditions for convergence of the procedure were found in terms of Lyapunov functions. They are applicable even to strongly nonlinear stochastic equations involving e.g. porous medium of p-Laplace operators. See J. Seidler, F. Žák: Electronic Communications in Probability 22, 36 (2017), 13 pp.
a6) A characterisation of the topological support of the law of a solution to a stochastic differential equation was found in terms of a modular Besov-Orlicz space. Compared to available results, the assumptions on the coefficients of the equation are

considerably relaxed and the description of the support is sharper. See M. Ondreját, P. Šimon, M. Kupsa, Stochastic Analysis and Applications 36, 6 (2018), pp. 1037-1052.
a7) The stochastic maximum principle for ergodic control problems was studied; in particular, necessary (and sufficient) conditions were found for optimality for controlled dissipative systems in finite dimensions. The approach based on the stochastic maximum principle requires much less regularity than the standard dynamic programming approach. See C. Orrieri, G. Tessitore, P. Veverka, Applied Mathematics and Optimization 79, 3 (2019), pp. 567-591.
Some of the results were obtained in cooperation with L. Baňas from Universität Bielefeld, A. de Bouard from Ecole Polytechnique, Z. Brzeźniak from the University of York, P. Čoupek and B. Maslowski from Charles University, E. Hausenblas from Montanuniversität Leoben, E. Motyl from the University of Łódź, M. Neklyudov from Università di Pisa, C. Orrieri from Sapienza Università di Roma, A. Prohl from Universität Tübingen, and G. Tessitore from Università di Milano. P. Šimon was a Master student supervised by M. Ondreját, F. Žák was then a student at Imperial College consulting with J. Seidler. The authors contributed equally to the respective papers.
b) Research in the field of interacting particle systems:
b1) *Convergence of discrete models to the Brownian net:* The Brownian web is a random continuum object that can loosely be thought of as a collection of coalecing one-dimensional Brownian paths, started from every point in space and time. The Brownian net is a variant of this where paths not only coalesce but also branch (separate) with a certain rate. The Brownian web (resp. net) is known to be the scaling limit of coalescing (resp. coalescing and branching) nearest-neighbour random walks. Since non-crossing properties play an important role in the construction of the Brownian web and net, convergence is easier to prove for nearest-neighbour random walks, which share these non-crossing properties. Convergence for non nearest-neighbour random walks has so far only been proved in the case of the Brownian web. Nevertheless, important steps towards establishing an analogous result for the Brownian net were made in the paper R. Sun, J.M. Swart, J. Yu, Annals of Applied Probability 29, 4 (2019), 2556-2593. This paper studies branching and coalescing random walks by means of their dual, the biased voter model. In particular, it establishes interface tightness for biased voter models, which is of independent interest.
b2) *Rank-based particle systems:* A number of authors, motivated by various problems in real life, have introduced stochastic particle models where particles interact only through their relative order, usually in the sense that in certain situations, the highest (or lowest) particle is removed. This includes e.g. the Stigler-Luckock model in financial mathematics (execution of the best buy/ask offer). In the paper J.M. Swart, Annals of Applied Probability 28, 3 (2018), pp. 1491-1535, some formulas for the Stigler-Luckock model were proved that had been derived in a nonrigous way in the financial literature. The paper V. Peržina, J.M. Swart, Journal of Applied Probability 55, 3 (2018), pp. 667-681, treats an extension of the model. The paper J.M. Swart, Markov Processes and Related Fields, 23, 1 (2017), pp. 87-102, treats a model loosely inspired by canyon formation. In M. Formentin, J.M. Swart, Alea 13, 2 (2016), pp. 1151-1164, a precise asymptotic result is proved for a queueing model (execution of the most urgent task) that is particularly amenable to analysis.
b3) *Recursive tree processes:* A Recursive Tree Process (RTP) is a generalisation of a Markov chain where time has a tree-like structure and flows in the direction of the root. More precisely, an RTP consists of a regular rooted tree plus an i.i.d. collection

of maps attached to the vertices of the tree that describe the state of that vertex as a function of the states of its descendants. Aldous and Bandyopadhyay (2005) defined an infinite, stationary RTP to be endogenous if the state of the root is a.s. determined by the random maps attached to the vertices. In T. Mach, A. Sturm, J.M. Swart, Mathematical Physics, Analysis and Geometry 21, 4 (2018), paper no. 30, 19 pp., we give a new characterisation of endogeny and exploit its connection to the convex stochastic order.

b4) *Markov process duality:* Duality is an important tool in the study of Markov processes. In the past, suitable duality function were usually found by ad hoc methods, but recently a number of authors have attempted a more systematic approach. In A. Sturm, J.M. Swart, Journal of Theoretical Probability 31, 2 (2018), pp. 932-983, we make a systematic study of pathwise dualities for monotone particle systems, unifying dualities introduced earlier by Gray, Krone, and others.

b5) Sharpness of phase transitions: It is known that the connection probability in percolation decays exponentially fast in the whole subcritical regime. This was first proved by Menshikov (1986) and then by Aizenman and Barsky (1987), who used Russo's formula and a system of two differential inequalities. Bezuidenhout and Grimmett (1991) adapted their methods to oriented percolation and the contact process. Duminil-Copin and Tassion (2015) gave a much simplified proof for both unoriented and oriented percolation that involves only a single differential inequality. In J.M. Swart, Probability Theory and Related Fields 170, 1 (2019), pp. 1-9, we gave a new, simple proof for the contact process that surprisingly does not use Russo's formula or differential inequalities but instead is based on a special harmonic function.

b6) *Cooperative branching:* In A. Sturm, J.M. Swart, Annals of Applied Probabability 25, 3 (2015), pp. 1616-1649, we study a particle system with cooperative branching and coalescence. This model is interesting because it exhibits a phase transition between extinction and survival while in the subcritical regime, extinction is slower than exponential, i.e., the spectral gap is zero. This contrasts with what is known for the contact process.

b7) *Generalisations of the zero-range process:* It is known that in zero-range processes, condensation can only occur when the process is non-attractive, which makes its study harder. In recent years, many authors have studied generalisations of the zero-range process. In L. Fajfrová, T. Gobron, E. Saada, Electronic Journal of Probability 21, 60 (2016), 52 pp., the authors make a very thorough study of one of the most general classes studied to date, which significantly for the first time yields an attractive process that exhibits condensation.

b8) M. Formentin, during his 6 months as a postdoc at ÚTIA in 2015, not only wrote the paper already mentioned in (b2), but also completed several papers with various coauthors based on his previous research in Italy. Let us mention in particular the paper A. Giometto, M. Formentin, A. Rinaldo, J. Cohen, A. Maritan, Proceedings of the National Academy of Sciences of the United States of America 112, 25 (2015), pp. 7755-7760, that studies Taylor's law, which is a scaling relation between mean and variance that has been observed in many population models. The authors argue that the observed exponent of many studies is a result of the sampling procedure and does not reflect the underlying population.

A large number of the papers mentioned above are written in cooperation with researchers from abroad, namely T. Gobron from Université de Cergy-Pontoise, E. Saada from Université Paris Descartes, A. Sturm from Universität Göttingen, R. Sun from the National University of Singapore, and J. Yu from New York University Shanghai. V. Peržina was a Master student of J. Swart, T. Mach a Ph.D student of A.

Sturm, but his paper cited in (b3) was completed when he was a postdoc in ÚTIA. In mathematics, is is customary that proofs arise as the result of joint discussions and all authors participate in writing the manuscript. In such a situation, all authors contribute approxiately equally to the result.

B. Our research in the field of ergodic theory resulted in two papers. The research on return times in low-complexity dynamical systems reveals the need to better understand the coding of a rotation of the unit circle with respect to a large class of partitions. We extended the well-known theory of coding the rotation of a circle with respect to decompositions into intervals to a significantly larger coding class, where each set of the partition can be a union of finitely many intervals. Unlike classical partitions, the newly described class is dense in the set of all measurable partitions. It allows to investigate standard properties and other objects, e.g. return times statistics, on a residual set of partitions. See M. Kupsa, Š. Starosta, Discrete and Continuous Dynamical Systems 35, 8 (2015), pp. 3483-3501. M. Kupsa's contribution to this paper amounts to 80 per cent. Further, topological properties of the set of invariant measures for a general hereditary shift were studied. We showed the arcwise-connectedness of this set with respect to the topology induced by the d-bar metrics for a large class of transformations. A consequence is the intermediate-value property for entropy understood as a function on the set. See J. Konieczny, M. Kupsa, D. Kwietniak, Proceedings of the American Mathematical Society 146, 8 (2018), pp. 3425-3438. All authors contributed equally to this paper.

C. In mathematical statistics, the research of the team has focused on multiple-output regression methods and statistical survival analysis.

c1) We extended two directional multiple-output quantile regression methods to the locally polynomial regression context, see the papers M. Hallin, Z. Lu, D. Paindevene, M. Šiman, Bernoulli 21, 3 (2015), pp. 1435-1466, and P. Boček, M. Šiman, Computational Statistics 32, 3 (2017), pp. 929-946. We implemented the methods and related functions in R and Octave/Matlab in two software packages (both can be freely downloaded, here and here). These packages are described in the papers P. Boček, M. Šiman, Kybernetika 53, 3 (2017), pp. 480-492, and Kybernetika 52, 1 (2016), pp. 28-51.

c2) We continued the research of elliptical multiple-output quantiles and extended them to general location and regression frameworks. For example, we demonstrated when these quantiles preserve the symmetry of the underlying distribution, proposed some model validation and diagnostic tools, and suggested how they can be computed or used for the classification of heteroscedasticity. See M. Hallin, M. Šiman, Statistics & Probability Letters 109, 1 (2016), pp. 232-237, and D. Hlubinka, M. Šiman, Test 24, 2 (2015), pp. 249-264.

c3) We reviewed existing multiple-output quantile regression methods for a prestigious handbook, see M. Hallin, M. Šiman in Handbook of Quantile Regression, Chapman and Hall/CRC 2017.

c4) In the field of statistical survival analysis, the team studied the generalisation of random censoring to a dependent competing risk scheme. Namely, conditions of identifiability of the full model (i.e. joint and marginal distributions) were explored, a rather complicated scheme with regression being considered, where also the corresponding copula parameter could depend on covariates. The problem of stochastic optimisation was studied in cases when the stochastic characteristics were estimated from incomplete data, and, in particular, from randomly right-censored data. See e.g. P. Volf, Croatian Operational Research Review 10, 1 (2019), pp. 13-21, or Kybernetika 54, 6 (2018), pp. 1156-1166.

Some of the results were obtained in cooperation with D. Hlubinka from Charles University, M. Hallin, D. Paindaveine (both from Université libre de Bruxelles) and Z. Lu from the University of Southampton. P. Boček in an IT expert of the team. All authors contributed approximately equally to the results.

D. In the field of statistical signal processing, the following problems were addressed:

d) Tensor decomposition methods (where by tensor we mean here a multidimensional linear array of a rectangular shape).

d1) We proposed a number of new algorithms for the canonical polyadic (CP) decomposition of tensors, which serves for storing tensors in a compressed form: first, a deflation algorithm, which allows parallel computing of one or several rank-one components of the CP decomposition, see IEEE Transactions on Signal Processing 63, 22 (2015), pp. 5924-5938 and pp. 5939-5950, second, a partitioned alternating least squares technique which is an improvement of the popular alternating least squares algorithm, see IEEE Signal Processing Letters 23, 7 (2016), pp. 993-997, and third, a novel second-order optimisation algorithm called the Krylov-Levenberg-Marquardt (KLM) algorithm, see IEEE Signal Processing Letters 26, 11 (2019), pp. 1653-1657.

d2) We proposed a new tensor diagonalisation technique suitable especially for block-term decomposition, see Signal Processing 138, 1 (2017), pp. 313-320

d3) We proposed a new technique called error preserving correction, which allows for a given approximate CP decomposition of a given tensor to find another approximate decomposition which has the same approximation error but better numerical properties, see IEEE Transactions on Signal Processing 67, 5 (2019), p. 1175-1190.

d4) It was shown how CP tensor decompositions can be used to find new algorithms for small matrix multiplication, see Journal of Computational and Applied Mathematics 317, 1 (2017), p. 362-370.

The results on tensor decompositions were obtained in cooperation with A. Cichocki and A.-H. Phan from the Riken Brain Science Institute in Wako, Japan. (They later moved to Skolkovo University of Technology, Moscow, Russian federation.) All authors contributed equally to the results.

e) Independent component analysis (ICA) and blind source separation.

e1) We studied the problem of extracting one independent component from an instanteneous linear mixture of signals. For this problem, we proposed a new formulation, and novel algorithms, see Z. Koldovský, P. Tichavský, IEEE Transactions on Signal Processing 67, 4 (2019), pp. 1050-1064.

e2) Together with our students Šembera and Kautský we proposed new algorithms for blind source separation of nonstationary sources modeled as piecewise AR(1) or piecewise non-Gaussian sources. The results have up to now been published in several proceedings papers.

e3) We summarized some of our previous results in this area in a book chapter, see Z. Koldovský, P. Tichavský in Independent Component Analysis and Learning Machines, Elsevier 2015.

The results on independent component analysis were obtained in cooperation with Zbyněk Koldovský from the Technical University of Liberec. In (e1) and (e3), Z. Koldovský has contributed 60 per cent and P. Tichavský 40 per cent, in (e2), contribution of P. Tichavský was about 20-30 per cent.

E. To conclude, let us stress the team still follows its traditional policy to encourage its members to publish only papers containing new and surprising ideas, not to maximize fashionable quantitative indicators seemingly measuring the quality of research.